Introduction

The cross-section of any process is obtained from the ratio $\sigma = N/\mathcal{L}$, where N is the number of events and $\mathcal{L}$ is the luminosity. The luminosity depends on machine and beam parameters. In LEP/SLC its most precise determination comes from the measurement of the event rate of the known process through the inverse relation $\mathcal{L} = N/\sigma_{\rm known}$. The *known* process should be calculable from well-established theory, and the low-angle Bhabha (LABH) $e^+e^- \rightarrow e^+e^-$ scattering process fulfils this role. Why? Because it is dominated by t-channel photon exchange and, in principle, can be calculated within perturbative Quantum Electrodymamics (QED) with arbitrary precision. It is also important that at sufficiently low angles the event rate for the LABH process is higher than the Z event rate at the top of Z resonance. In fact, the systematic component of the experimental error is now dominant. The experimental precision of the luminosity measurement at LEP has increased dramatically from the beginning of its operation. ¿From the initial 1.0% (already a big improvement over the 2–3% of PETRA/PEP) it has now gone below 0.1%, with a level of 0.05% not out of the question in the future! The 'theoretical uncertainty' in the calculation of the LABH process for realistic acceptance/cuts ($\simeq 0.25\%$) is now dominant. There is an urgent need to either decrease it to the level of the purely experimental one or better it. Is this possible? Most probably, yes.

In this brief introduction we will characterize the main features of the theoretical calculation of the LABH process. We will also touch on the question of the significance of the precise measurement of the luminosity for precision tests of the standard model.

The LABH cross-section is calculated from QED with the usual perturbative techniques. The matrix element is calculated from Feynman diagrams, and the integration over the phase space with cuts/acceptance corresponding to realistic experimental conditions is done numerically. The QED perturbative series is truncated to a certain order. Higher orders can also be estimated and summed up by certain standard techniques such as exponentiation. At PETRA/PEP $\mathcal{O}(\alpha^1)$ calculation was sufficient (see following sections for more information). Already at that time it was concluded that the phase-space integration for complicated realistic cuts/acceptance could only be done with the help of the Monte Carlo (MC) method – or more precisely, only by using a MC event generator.

Let us now briefly characterize the basic features of the QED perturbative calculations of the LABH cross-section. Let us start with 'photonic' corrections in which the Born diagram with a single t-chanel photon exchange is dressed with any number of virtual and real photon lines. First of all, the smallness of the electron mass squared compared with the typical t-channel transfer ($|t| \sim 1$ GeV^2) has to be stressed. As a result, the QED corrections are often magnified by the so-called big logarithm $L = \ln(|t|/m_e^2)$. At LEP, even for a scattering angle as low as $\theta = 0.025$, we have $L = 15$. At the same time, relevant formulae and kinematics simplify a lot because terms proportional to $m_e^2/|t|$ may be neglected. Terms of order $\theta^2 \simeq 4|t|/s$ are also often small enough to be neglected, especially at angles below 50 mrad. They usually have to be kept at the Born cross-section and in kinematics. Another useful feature of QED corrections to the LABH process is the possibility of neglecting an entire class of corrections named as up–down interference. This is a subset of QED real and virtual corrections in which, in addition to the usual t-channel photon, other photon lines connect the upper e^- line with the lower e^+ fermion line. At angles small enough ($\theta < 100$mrad) and for the usual experimental cuts the total —virtual plus real— contribution from all corresponding Feynman diagrams is negligible (see contribution by Jadach et al.) for a more extensive discussion).

In general, pure 'photonic' corrections are difficult to calculate but can be made as small as possible by adding higher orders in the calculations. The other 'non-photonic' corrections are dominated by the vacuum polarization and $\gamma - Z$ interference. Also, light fermion pair production and multiperipheral-type diagrams enter at the $\mathcal{O}(\alpha^2)$ (see, for instance, contribition by Arbuzov et al.) The vacuum polarization on the t-channel photon line is as sizeable as the bulk of 'photonic' corrections. It is inherently uncertain because of the QCD component, calculated using a dispersion relation which has as input the low-energy experimental data for the cross-section of $e^+e^- \rightarrow q\bar{q}$. Only better experimental data or better analysis of existing data will allow this error to be reduced substantially (see also contribution by Beenakker et al.) The interference of t-channel γ_t and s-channel Z_s is a kind of 'pollution' of the LABH process. This interference is small: below 0.5% for the second generation of the luminosity detectors located at lower angles. Unfortunately, it is not small enough to be treated in the Born approximation. The QED 'photonic' corrections to this contribution are important because it varies strongly with the total center-of-mass energy. Furthermore, it is also affected by the up–down interferences. (see contributions by Beenakker et al. and ? for more on this). Light fermion pair $(ee, \mu\mu)$-production and multiperipheral-type diagrams most probably contribute very little ($<$ 0.1%) but have to be examined quantitatively at the present experimental precision level of $\simeq 0.1\%$.

What is the significance of the precise measurement of the luminosity, and therefore of the absolute cross-section for the program of the precision tests of the standard electroweak model (SEM) at LEP? As is well known, the measurement of the invisible width of Z conveniently parametrized in the number of massless neutrinos N_ν is affected directly by $\delta\mathcal{L}/\mathcal{L}$. The experimental deviation $N_\nu - 3$ would signal very important non-standard physics (a new channel of Z decay or deviation of neutrino coupling constants from the SEM). It is also well known that $N_\nu - 3$ and $\sigma_{\rm tot}(M_Z)$ have very little sensitivity to details of the SEM such as the masses of the Higgs boson and top quark. The error of the luminosity measurement also affects the measurement of the Z decay width into electrons Γ_e. This quantity is one of the main sources (together with asymmetries) of our knowledge of the electroweak mixing angle.

In the sections that follow, the reader will find further details on the present status of the theoretical calculations of the LEP/SLC luminosity cross-section.

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