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HIGHER-ORDER RADIATIVE CORRECTIONS TO BHABHA SCATTERING AT LOW ANGLES: YFS MONTE CARLO APPROACH*

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ABSTRACT

In this contribution we present new results on the QED second-order radiative corrections to the low-angle Bhabha cross section. The presented results will be essential in the future reduction of the overall theoretical uncertainty in the measurement of the luminosity at LEP below the present 0.25% level.

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1. Introduction

The experimental precision of the luminosity measurement has increased dramatically from the times of PETRA (2%) and early LEP (0.5%) down to the present value close to 0.1%. It is measured with the help of the low-angle Bhabha (LABH) process $e^+e^- \to e^+e^-$. In the LEP/SLC accelerators the LABH process is measured in the angular range below 100 mrad. The luminosity determined in this way provides absolute normalization of the cross section of all other processes in the e^+e^- scattering. The LABH cross section is therefore not of physical interest itself, but, on the contrary, is regarded as completely known from theory, i.e. from Quantum Electrodynamics (QED). On the other hand, although in principle the LABH cross section is calculable in perturbative QED with arbitrary precision (except a small hadronic correction), it is subject to theoretical uncertainties due to a truncation of the perturbative expansion and also due to a limitation of the calculational tools (computer programs). All LEP/SLC experiments use theoretical calculations for LABH, based on works published by some of the actual authors three years ago[1]. This calculation has an overall theoretical/technical precision precision of 0.25% and is embodied in the form of the Monte Carlo (MC) event generator[2] BHLUMI version 2.0. This error was acceptable in 1991 but now, with an improvement of the experimental precision by a factor of two more it dominates the present overall luminosity error. It is therefore quite urgent to reduce the theoretical error of the QED calculation down to a precision level of 0.1% at least.

The backbone of the 0.25% theoretical precision estimate[1] is due to missing second-order $\mathcal{O}(\alpha^2 L^2)$ (0.15%) and $\mathcal{O}(\alpha^2 L)$ (0.09%) contributions in the matrix element encoded in the Monte Carlo calculations. Here $L = \ln(|t|/m_e^2)$ is the so-called big-log in the leading-logarithmic (LL) approximation where t is t-channel transfer (of order 1 GeV); see also Fig. 1 for a pictorial definition of the LL approximation.

The first of the above contributions (0.15%) includes also the *technical precision* of the Monte Carlo programs due to programming bugs, rounding errors, quality of random numbers, etc. It is illustrated[1] in Fig. 2 as a difference of three Monte Carlo sub-generators of BHLUMI 2.0: (i) multiphoton $\mathcal{O}(\alpha)_{exp}$ BHLUMI, (ii) $\mathcal{O}(\alpha)$ OLDBIS (without exponentiation) and, (iii) $\mathcal{O}(\alpha^3)_{LL}$ leading logarithmic (collinear photon emission) event subgenerator LUMLOG. The difference of the three MC subgenerators provides a solid estimate of the technical precision. In addition, subgenerators (ii) and (iii) have separate estimates of their technical precision at the level below **0.05%** coming, in the case of (ii), from an independent comparison[3] with a semi-analytical calculation, and, in the case of (iii), with another Monte Carlo[4]. The comparison in Fig. 2 provides, therefore, an estimate of the technical precision mainly for the multiphoton BHLUMI subgenerator, which did not have any other independent analytical or Monte Carlo cross-check[†]

As for the *physical precision*, which is mainly due to a truncation of perturba-

[†] At the time it seemed unthinkable to integrate analytically the total cross section of the $\mathcal{O}(\alpha)_{exp}$ BHLUMI.

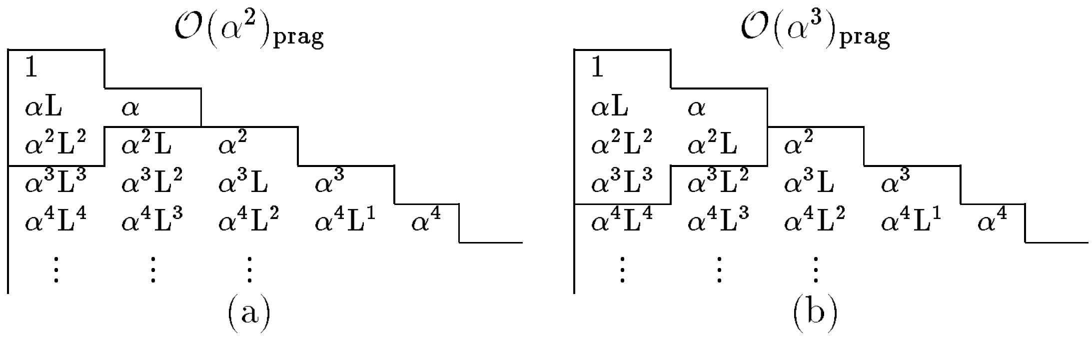


Figure 1: QED perturbative leading and subleading corrections. Rows represent corrections in consecutive perturbative orders – the first row is the Born contribution. The first column represents leading logarithmic (LL) approximation and the second column depicts the next-to-leading (NLL) approximation. In the figure, terms selected for (a) second and (b) third-order pragmatic expansion are limited with help of an additional line.

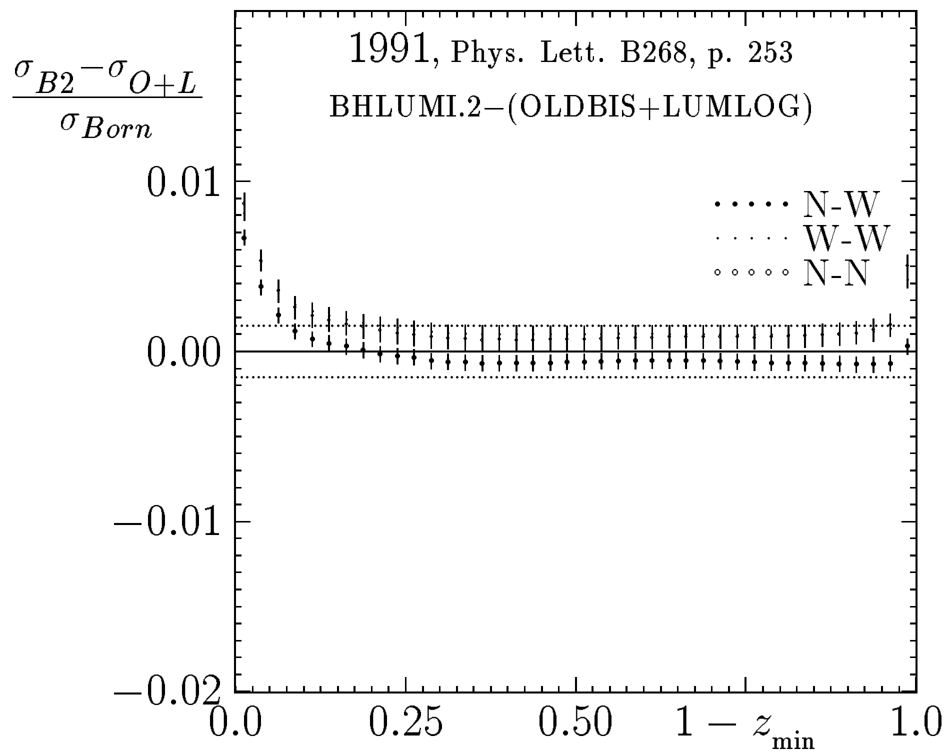


Figure 2: We plot the difference[1] of σ_{B2} of BHLUMI 2.0 and σ_{O+L} from OLDBIS and LUMLOG. It represents the missing $\mathcal{O}(\alpha^2 L^2)$ bremsstrahlung correction in BHLUMI version 2.0 event generator[2] together with its technical precision. The difference of the cross sections (divided by Born) is calculated for symmetric and asymmetric calorimetric trigger Ξ_{NW} as a function of the energy cut $z_{\rm min}$. Dotted lines mark the 0.15% limit. Vacuum polarization, Z and s channel γ are switched off.

tive calculation[1], the dominant (beyond first-order) correction of $\mathcal{O}(\alpha^2 L^2)$ was under good control because it was calculated using the $\mathcal{O}(\alpha^3)_{LL}$ subgenerator LUMLOG[4]. The hybrid Monte Carlo calculation OLDBIS + LUMLOG includes the entire $\mathcal{O}(\alpha^2 L^2)$ correction for the integrated cross section, but due to zero-angle collinear emission of photons, LUMLOG is not very suitable for experimental analysis where various fine-grain inclusive/muliphoton distributions are checked in the process of reducing the systematic experimental error. In view of the above, experimentalists have always preferred to use the muliphoton MC generator BHLUMI 2.0, which includes only the part of $\mathcal{O}(\alpha^2 L^2)$ generated by a Yennie-Frautschi-Suura exponentiation (providing excellent realistic differential distributions), and then to employ the OLDBIS+LUMLOG hybrid solution in order to estimate the missing $\mathcal{O}(\alpha^2 L^2)$ correction. For realistic cuts this correction has turned out to be small, typically below 0.2%.

The obvious development path of the above calculation scheme was the following: (A) to implement the $\mathcal{O}(\alpha^2 L^2)$ missing part of the matrix element in the muliphoton exponentiated subgenerator of BHLUMI, (B) to provide a new independent analytical cross-check of the new matrix element, (C) to improve the estimate of the next dominant bremsstrahlung-type corrections, i.e. of $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^3)$ corrections and, (D) to estimate again other higher-order corrections like light pairs, vacuum polarization, remnant of s-channel Z-exchange etc.

The outline of our talk is the following. In Section 2 we discuss step (A) and in Section 3 step (B). Step (C) will be mainly discussed in Section 4, but some preliminaries will be given in Section 3. Point (D), and to some extent also point (C), were already elaborated[5] in the literature.

Before we come to more details let us explain/define several useful concepts, approximations and terminology typical for QED calculations of the LABH cross section, which will be used or referred to in the course of this presentation.

Up-down interference: At angles below 100 mrad all "photonic" corrections in which the additional photon line connects the upper electron line with the lower positron line (so called up-down interference) are strongly suppressed. This phenomenon was conjectured and proved numerically, using an $\mathcal{O}(\alpha)$ calculation[3]. This phenomenon we call "suppression of the up-down interference". In all presented calculations we exploit this approximation.

The Monte Carlo (MC) technique which is used heavily in all presented work is nothing more and nothing less but the technique of the exact (up to statistical error) integration over the multiparticle phase-space.

Yennie-Frautschi-Suura (YFS) exponentiation is, in one word, a technique of summing exactly all infrared singularities to infinite order, which provides us with *exclusive* multiphoton differential distributions. The multiphoton distributions (with virtual corrections as well) are derived from Feynman diagrams and are calculated/improved orded by order.

Trigger is our short-hand name for the set of kinematical cuts which define accepted events for the LABH total cross section. Real experimental triggers of LEP/SLC detectors are *calorimetric*, i.e. all photons and electrons which satisfy the

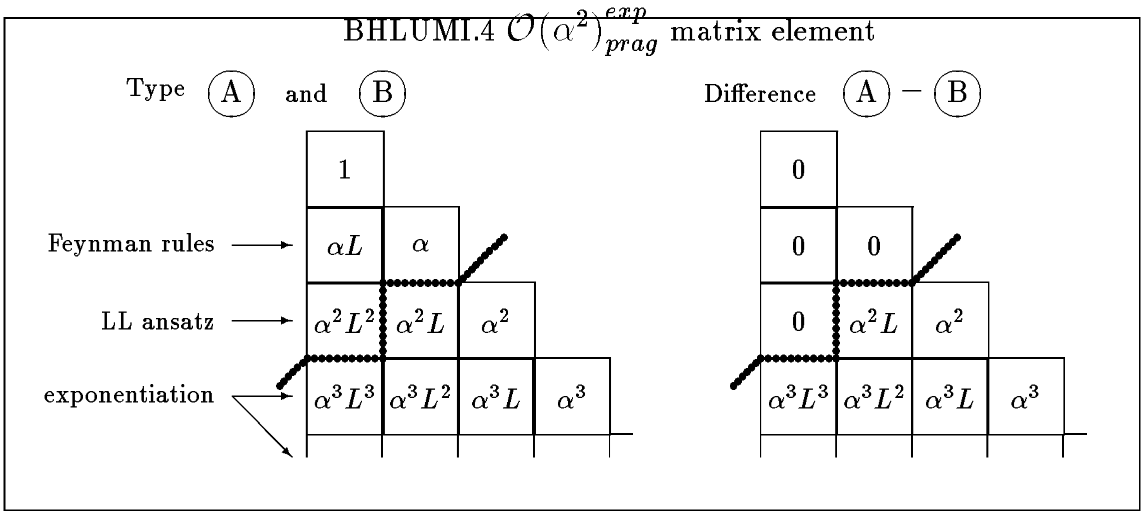


Figure 3: **Perturbative content of the matrix element of BHLUMI version 4.0 in the pictorial representation. The correct/complete contributions of $\mathcal{O}(\alpha^2)_{prag}$ are above dotted line. The right hand side picture illustrates perturbative content of the difference of the two types A and B of the $\mathcal{O}(\alpha^2)_{prag}$ matrix elements.**

angular acceptance condition $\theta_{\min} < \theta < \theta_{\max}$ are registered without making any distinction among them, and the minimum total energy required is $x = E/E_{beam} > 1 - X_{\max}$ in the forward and backward hemisphere simultaneously. In practice, quite often $\theta_{\min,\max}$ in the forward and backward direction are taken different (asymmetric trigger). Also, in the real experiment the association of photons and electrons into a single cluster with energy E and average angle θ is a little bit involved, see contribution of B. Pietrzyk[6] for more details. In the calculations we often use, as an example, the algorithm which describes the trigger of the new ALEPH SICAL detector.

2. New $\mathcal{O}(\alpha^2)_{prag}$ exponentiated BHLUMI.4 Monte Carlo

In the following we shall characterize the new $\mathcal{O}(\alpha^2)_{prag}$ exponentiated matrix element implemented in the new version of Monte Carlo BHLUMI.4.0. We cannot afford to include here the full definition of the matrix element used in the program because of lack of space (it requires more than five pages). Nevertheless we shall try to characterize all its essential properties.

In the $\mathcal{O}(\alpha^2)$, in order to reach 0.1% physical precision level, it is probably enough to add in the Monte Carlo matrix element, beyond the regular $\mathcal{O}(\alpha)$, the dominant second-order contribution of $\mathcal{O}(\alpha^2 L^2)$ (if possible, with exponentiation). This type of calculation we denote as $\mathcal{O}(\alpha^2)_{prag}$ and it is depicted in Fig. 1a and in Fig. 3. In fact, in BHLUMI.4 we include even two examples of the $\mathcal{O}(\alpha^2)_{prag}$ matrix element, marked with A and B, which differ by $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^2)$ terms. This is also illustrated in Fig. 3. Why are we free to pick two choices and why is it

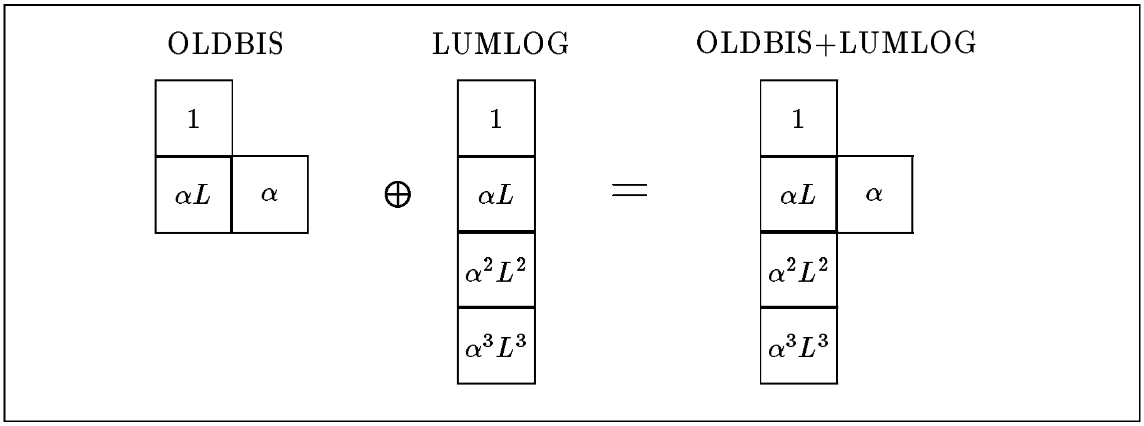


Figure 4: **Perturbative content of the matrix element of the hybrid Monte Carlo calculation OLDBIS+LUMLOG.**

profitable to do it? While $\mathcal{O}(\alpha)$ distributions, come directly from the Feynman diagrams (no freedom!) the additional $\mathcal{O}(\alpha^2 L^2)$ contributions we derive more simply by convoluting twice the Altarelli-Parisi kernel. The same kind of LL ansatz was used successfully in the YFS2 and YFS3 Monte Carlo programs [7,8,9]. For the $\mathcal{O}(\alpha^2 L^2)$ contribution, the soft limit is improved by hand‡to the well known behaviour and the finite transverse momenta of photons are introduced in the distributions using the soft limit as a model to follow. Obviously, the above procedure has some freedom in the construction of the matrix element. How big is the freedom? Let us first note that (even without exponentiation!) the above procedure creates some *non-zero* contributions of $\mathcal{O}(\alpha^2 L^1)$ and $\mathcal{O}(\alpha^2 L^0)$. The two types of $\mathcal{O}(\alpha^2)_{prag}$ matrix elements may have different such contributions. The important advantage of our $\mathcal{O}(\alpha^2 L^2)$ ansatz is that it is simple (also quick in the computer evaluation) and its LL content, which is of primary interest, is explicit and therefore very easy to control. As we have already mentioned, the LL ansatz for the $\mathcal{O}(\alpha^2 L^2)$ comes before YFS exponentiation. Exponentiation introduces new non-zero contributions of $\mathcal{O}(\alpha^3)$ and higher-orders. Among them, the $\mathcal{O}(\alpha^3 L^3)$ contribution will be numerically dominant. Since YFS exponentiation is well founded physically, these higher-order terms improve perturbative convergence of the calculation substantially. (This was proved explicitly for the $\mathcal{O}(\alpha^3 L^3)$ terms[10].)

Note that option B for matrix element, degraded to $\mathcal{O}(\alpha)_{prag}$ exponentiated, is identical to the matrix element in the published BHLUMI 2 program. The BHLUMI.4 is also backward compatible with BHLUMI.2 for OLDBIS and LUMLOG subgenerators. They are still included. In fact the LUMLOG generator with LL matrix element (exponentiated and unexponentiated) up to third-order is extended a little bit, because emission of photons is now included not only in the initial state but also in the final state. It is done in the zero transverse momentum approximation as previously. For completeness we depict the perturbative content of the

‡**In the LL approximation correct soft limit is in the general case not reproduced.**

LUMLOG and OLDBIS event generators in Fig. 4.

3. Technical precision

Generally, technical precision is obtained by doing two technically very different calculations and taking the difference. The most powerful method is to take difference between the Monte Carlo and analytical calculation[4]. The serious disadvantage of this method is that it can be applied only for certain rather simple kinds of trigger. In the case of BHLUMI.4 even the existence of one such trigger for which the above method can be applied is a highly non-trivial question! The method in which two different Monte Carlo calculations are compared can be applied for a wider family of cuts. Its very serious disadvantage is that if one encounters (it is always the case!) an intolerably big difference of the two MC results then it is very difficult, often impossible, to find the source of the difference (debug the corresponding MC programs).

We are in the process of determining the technical precision of BHLUMI.4 using an elaborate multistep method. It consists of the following steps:

1. Invent "academic trigger" for which *analytical* integration of the BHLUMI cross section is feasible down to a precision of 3×10^{-4}. This precision level requires a calculation of $\mathcal{O}(\alpha^3)_{prag}$.

2. Perform analytical calculation and debug the BHLUMI.4 Monte Carlo program and the corresponding semi-analytical calculation/program until $|\sigma_{MC} - \sigma_{analyt}| < 3 \times 10^{-4}$ is obtained for the "academic trigger".

3. Do the same for the easier cases of OLDBIS and LUMLOG.

4. Take the difference BHLUMI − (OLDBIS + LUMLOG) and explore it analytically and numerically in every fine detail down to 3×10^{-4} precision.

5. Do an "adiabatic transition" from academic trigger to realistic trigger using series of intermediate triggers looking carefully at the evolution of the difference BHLUMI − (OLDBIS + LUMLOG). It should be understood to a precision comparable to 3×10^{-4} (for instance better than 5×10^{-4}).

In the above scenario we take advantage of the extremely important fact that the technical precision for (OLDBIS + LUMLOG) is practically zero for any kind of trigger!

In the following we shall show results concerning step 1 in the method outlined above, i.e. we shall: (a) define a set of "academic" cuts used for the semi-analytical integration of the above matrix element over muliphoton phase-space, (b) briefly characterize methods used in the analytical integration and the class of corrections kept in the analytical phase-space integration (it is not the same as in matrix element!), (c) show the numerical agreement of the Monte Carlo (with the new matrix element) with the semi-analytical formula down to the 0.03% level (technical precision).

Ad (a): The most important criterion used to define our set of kinematical cuts for semi-analytical integration of the $\mathcal{O}(\alpha^2)_{prag}$ new matrix element over the multiphoton phase-space (so called *academic trigger*) is that, actually, this semi-analytical integration is really feasible. We define the cuts of our "academic trigger" as follows: $|t_{min}| < |t| < |t_{max}|$ and $V < V_{max}$, where t is the four-momentum transfer squared and the variable V represents some kind of measure of the total energy carried away by all emitted real photons. We require that $0 < V < 1$ represents the condition of completeness of the phase-space and $V < \epsilon$ represents the condition that all photons are soft. The V-variable we actually define, in terms of the four-momenta, as follows:

$$V = 1 - \frac{2(p_1 p_2)\ |t|}{(2(p_1 p_2)\ + 2(p_1 K_p))^2}\ \frac{2(q_1 q_2)\ |t|}{(2(q_1 q_2)\ + 2(q_1 K_q))^2}, \tag{1}$$

where $p_i = 1, 2$ are the four-momenta of the incoming and outgoing electron, $q_i = 1, 2$ are the four-momenta of the incoming and outgoing positron and, K_p and K_q are the total four-momenta of all photons emitted from electron and positron lines, respectively.

Ad (b): With the above definition of the phase-space window, it is rather straightforward to integrate $\mathcal{O}(\alpha^2)_{prag}$ matrix element, keeping all terms within the $\mathcal{O}(\alpha^2)_{prag}$ approximation. This we found ***insufficient*** for the purpose of establishing a technical precision at the 0.03% level because some terms beyond $\mathcal{O}(\alpha^2)_{prag}$ (especially for partial incomplete results!) are of that order. We have therefore decided to follow in the integration the $\mathcal{O}(\alpha^3)_{prag}$ approximation; see also Fig. 1b. It means that terms of $\mathcal{O}(\alpha^2 L)$ due to our LL ansatz and terms of $\mathcal{O}(\alpha^3 L^3)$ due to exponentiation are integrated analytically over the phase-space (with academic trigger) exactly!

The resulting integrated cross section is not very complicated and it reads as follows:

$$\begin{aligned}
&\sigma_B^{(2)}(t_{min}, t_{max}, V_{max}) = \int_{t_{min}}^{t_{max}} dt \int_0^{V_{max}} dV\ \ \rho_{tot}^{(2)}(t, V)\\
&\rho_{tot}^{(2)}(t, V) = b_0\ F(2\gamma)\ e^{2\Delta_{YFS}(\gamma)}\ 2\gamma V^{2\gamma-1}\Big\{1 + \gamma + \gamma^2/2\Big\}\\
&\quad +b_0\ F(2\gamma)\ e^{2\Delta_{YFS}(\gamma)}\ V^{2\gamma}\Big\{\gamma(-2+V) + \frac{\alpha}{\pi}\ln(1-V)(-4+4V-2V^{-1})\\
&\quad +\gamma^2(-2) + \gamma^2\ln(1-V)(3 - 3V/2 - 2V^{-1})\\
&\quad +\gamma^3(-7V/4) + \gamma^3\ln(1-V)[5/4 + V/2 - 2V^{-1}]\\
&\quad +\gamma^3\ln(1-V)^2[-5/8 + 5V/16 + (1/4)V^{-1}] + \gamma^3\mathrm{Li}_2(V)(2-V)\\
&\quad +\gamma\frac{\alpha}{\pi}[1/4 + 11V\\
&\qquad\qquad -(13/4)(2-V)^{-1} + (1/2)(2-V)^{-2} - 6(2-V)^{-3} + 2(1-V)^{1/2}]
\end{aligned}$$

$$
\begin{aligned}
&+\gamma\frac{\alpha}{\pi}\ln(1-V)[39/4-19V/4-2V^{-1}\\
&\qquad -2(2-V)^{-1}+(2-V)^{-2}-(1/2)(2-V)^{-3}-(3/2)(1-V)^{1/2}]\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V/2)[-9/2+3V/4-4(2-V)^{-1}+2(2-V)^{-2}-4(2-V)^{-3}]\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V)^2[19/8-41V/16+V^{-1}]+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(2-V)(-1/2+V/4)\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(V)(12-10V)+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln(V/2)(-6+5V)\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V)\ln[1-(1-V)^{1/2}](-6+5V)\\
&+\gamma\frac{\alpha}{\pi}\ln(2-V)\ln(1-V/2)(3/2-11V/4)\\
&+\gamma\frac{\alpha}{\pi}\ln(1-V/2)^2(3/4-5V/8)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(1/2)(-3/2+11V/4)\\
&+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2[(1-V)/(2-V)](1/2-V/4)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(1/(2-V))(1-5V/2)\\
&+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2(-V/(2(1-V)))(6-5V)+\gamma\frac{\alpha}{\pi}\mathrm{Li}_2[1-(1-V)^{-1/2}](6-5V)\\
&-\xi\gamma\chi(V)/(1-V)\Big\}
\end{aligned}
\tag{2}
$$

where $\gamma = 2(\alpha/\pi)(L-1)$, $b_0 = \chi(\xi)$, $\chi(x) \equiv (1+(1-x)^2)/2$, $\xi = |t|/s$, $F(x) \equiv \exp(-Cx)/\Gamma(1+x)$, and $\Delta_{YFS}(\gamma) = \gamma/4-(\alpha/\pi)(1/2+\pi^2/6)$. Note that the $\mathcal{O}(\alpha^2)_{prag}$ part of the formula is very compact and that its LL content is identical to the non-singlet second-order structure function of the photon in the electron [§] The terms of $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L)$ which represent most of the formula, finally turn out to be numerically small, in fact below 0.04%.

Ad (c): In Fig. 5a we show a comparison of eq. (2) with the Monte Carlo BHLUMI.4. Although the integration over $|t|$ and V is feasible analytically we do it numerically with the help of the standard Gauss technique, because the integrand is a very smooth function, suitable for this method (peaks are removed either by change of variables or subtraction). As we see, the Monte Carlo and semi-analytical result differ up to 0.03%. We conclude that for the above "academic trigger" we have obtained 0.03% technical precision, i.e. we have attained the goals of steps 1 and 2.

In the multistep procedure outlined above we have gone through steps 1 to 4 and we have the first numerical results concerning step 5. In step 3 we have also obtained the simple semi-analytical results $\sigma_O^{(1)}$ and $\sigma_L^{(3)}$, which agree with the corresponding Monte Carlo OLDBIS and LUMLOG for "academic trigger" to better than 0.02%! The next numerical result shown in Fig. 5b is relevant for step 4 and it demonstrates that this step is also completed. As we see, the difference BHLUMI.4 − (OLDBIS + LUMLOG) is under control down to 0.05% because Monet Carlo and analytical results agree at this precision level. In the next Section we include more discussion

[§] **This is due to the fact the variable V in the LL has a very simple meaning.**

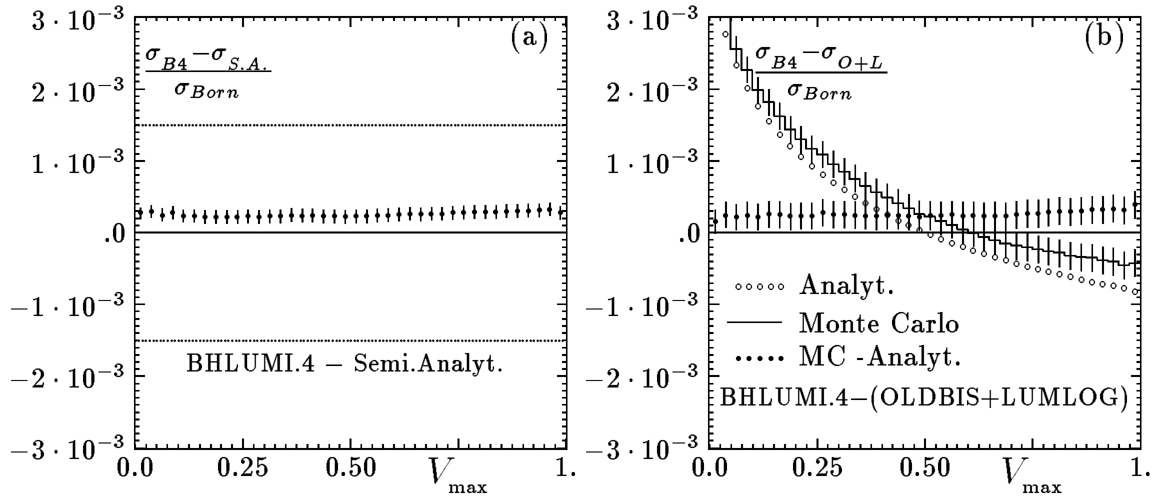


Figure 5: In (a) we show comparison of BHLUMI version 4.0 (unpublished) with semi-analytical formula for "academic" trigger defined with $|t_{\rm min}| < |t| < |t_{\rm max}|$ and $V < V_{max}$. Dotted lines mark the 0.15% limit. In (a) we demonstrate the some kind of comparison, analytical versus Monte Carlo, for the difference of the of the cross section (divided by Born) from three MC calculations BHLUMI.4 − (OLDBIS + LUMLOG). The limits $t_{\rm min,max}$ correspond to an angular range $26.125 < \theta < 55.875$ *mrad*.

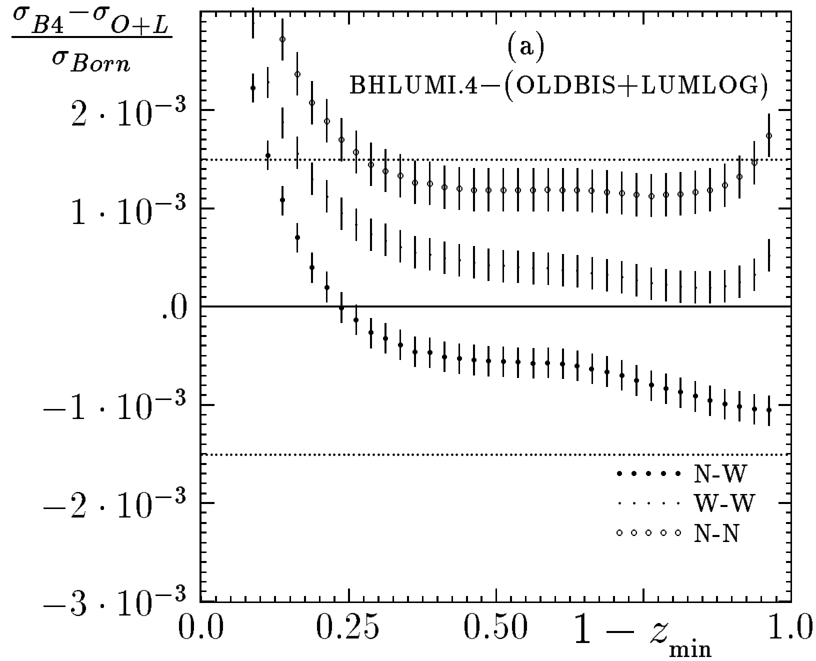
on the result of Fig. 5b.

The crucial question now is whether we can extend this result to triggers other than our "academic trigger", in particular if we can port our result to realistic experimental triggers. This part (step 5) is still being developed and we cannot show all relevant partial results due to lack of space.

4. Physical precision

The precision estimate in our previous work[1] was based to a large extent on the calculation of the difference BHLUMI.2 − (OLDBIS + LUMLOG) which, for BHLUMI.2, represented missing $\mathcal{O}(\alpha^2 L^2)$ plus technical precision. With the new matrix element in BHLUMI.4, which includes the complete $\mathcal{O}(\alpha^2 L^2)$ contribution we look immediately into the same three-generator difference. The result of such a comparison, with the scale on the vertical axis inflated by a factor of almost ten, is presented in Fig. 6. It is done for the same trigger type and angular range as in the older publication[1] (calorimeter of ELCAL/ALEPH type[1]) and also for new ALEPH/SICAL-type detector at lower angular range.

At first sight, the new result in Fig. 6 looks completely compatible with the old published result shown in Fig. 2. As we see in Fig. 6 the difference is again well within 0.15%. Of course, the interpretation of this difference is not the same now as in the previous work[1]. The muliphoton Monte Carlo BHLUMI.4.0 includes $\mathcal{O}(\alpha^2 L^2)$ corrections, hence the plotted difference BHLUMI.4 − (OLDBIS + LUMLOG) is



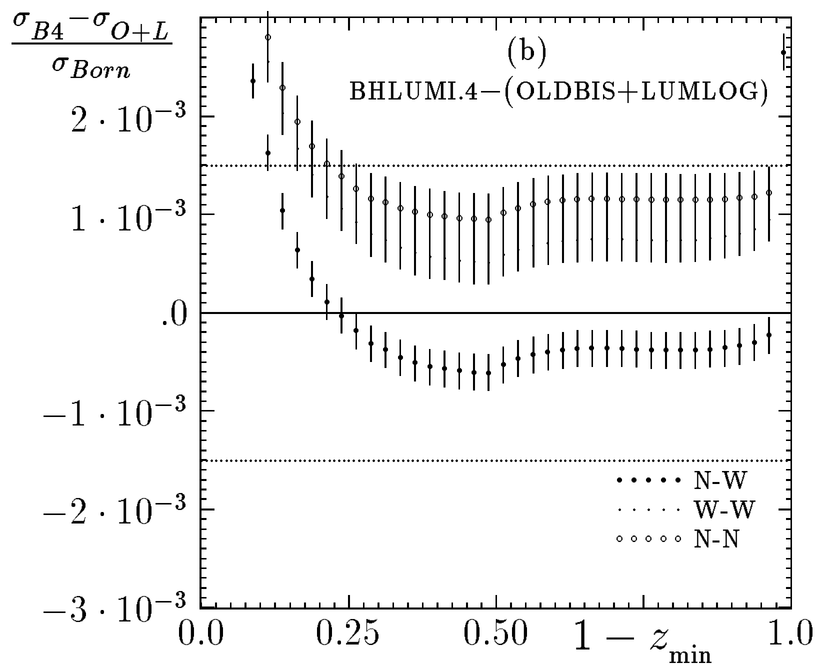


Figure 6: We plot the difference BHLUMI.4−(OLDBIS+LUMLOG) (a) for ALEPH/ELCAL type calorimeter/trigger (angular range $60-120\ mrad$) as defined in old publication[1] (b) for ALEPH/SICAL detector ($25-60\ mrads$). Dotted lines mark the same 0.15% limit as in Fig. 2.

potentially dominated by the $\mathcal{O}(\alpha^2 L)$, $\mathcal{O}(\alpha^3 L^3)$ and technical precision. (The estimate of the technical precision from Fig. 5 does not apply here automatically, due to the different type of trigger.) The $\mathcal{O}(\alpha^2 L^2)$ is absent in the new results of Fig. 6! The above new results are very encouraging but they should be treated as *preliminary* – they will soon be subjected to a new round of tests.

In the following we shall present additional numerical results which will teach us to better understand the meaning of the results in Fig. 6. The immediate question to answer is: How big was actually the "missing $\mathcal{O}(\alpha^2 L^2)$" correction in the published version of BHLUMI.2. We may simply look at the difference BHLUMI.4−BHLUMI.2. Since the published BHLUMI.2 includes the matrix element of type A we are examining the difference $\mathcal{O}(\alpha^2)^{exp}_{prag,A} - \mathcal{O}(\alpha^1)^{exp}_{prag,A}$. It is plotted in Fig. 7a for the SICAL detector of ALEPH. We see that the result is small in comparisons with typical values/estimates 0.25%–0.5% quoted in previous papers[4,1] for the $\mathcal{O}(\alpha^2 L^2)$ corrections. We simply conclude that the choice of the matrix element in BHLUMI.2 was very lucky! To see it more clearly, we show a similar quantity for the matrix element of type B in Fig. 7b. Here, the situation looks more "normal". The missing $\mathcal{O}(\alpha^2 L^2)$ contribution for the experimentally relevant $X_{\max} \simeq 0.5$ is -0.25%. Of course, the next logical question is: Do the two $\mathcal{O}(\alpha^2)_{prag}$ results of type A and B agree? Yes, they do, as we see in Fig. 7c the corresponding difference is tiny indeed, it is only $\simeq$ **0.01%**!

All the above cross-checks, together with the results from the previous Section give us strong hint that the new $\mathcal{O}(\alpha^2)_{prag}$ matrix element in BHLUMI.4 is correctly implemented. Nevertheless, since the problem of high technical precision for a realistic trigger is still not solved, we say that the difference BHLUMI.4 − (OLDBIS + LUMLOG) in the plots of Fig. 6 represent missing $\mathcal{O}(\alpha^3 L^3)$, $\mathcal{O}(\alpha^2 L)$ and the technical precision. In fact the $\mathcal{O}(\alpha^3 L^3)$ can be practically eliminated from this list. Using the semi-analytical formula (2) we can calculate *exactly* what the missing $\mathcal{O}(\alpha^3 L^3)$ is. The $\mathcal{O}(\alpha^3 L^3)$ missing terms are known[10] and can be easily included in eq. (2)! The effect of such a modification on the cross-section is shown in Fig. 7c. It is negligible, below 0.02%. This result is not that surprising, because it was shown[10] that the YFS exponentiation sums up the LL part of higher-orders very efficiently. (We suspect that it may not hold for $\mathcal{O}(\alpha^2)$ calculation without YFS exponentiation.) Note that the above result extends for any calorimetric experimental trigger because it is of a pure LL character. We conclude, therefore, that Fig. 6 contains only the missing $\mathcal{O}(\alpha^2 L)$ and technical precision.

Can we, in view of the above discussion, already improve on the total precision of the luminosity cross section? First of all, in the previous work[1] we estimated that the $\mathcal{O}(\alpha^2 L)$ contribution is generically of order 0.1%. In fact the value 0.09% was used and it was summed up with the 0.15% estimate of the missing $\mathcal{O}(\alpha^2 L^2)$ from Fig. 2 to obtain a total bremsstrahlung error of 0.24%. Since we now know that missing $\mathcal{O}(\alpha^2 L^2)$ was absent in the old Fig. 2 by luck and in the new Fig. 6 by construction, these figures show us $\mathcal{O}(\alpha^2 L)$ and technical precision. The 0.15% spread of the curves in Fig. 6 is very well compatible with the generic estimate of 0.1%. But since the generic estimate and the spread of the curves represent

Canonical coefficients				
	non-calorimetric case		calorimetric case	
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi}4L$	140×10^{-3}	$\xrightarrow{1/2}$	70×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.5×10^{-3}	$\xrightarrow{1}$	2.5×10^{-3}
$\mathcal{O}(\alpha^2 L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	10×10^{-3}	$\xrightarrow{1/4}$	2.5×10^{-3}
$\mathcal{O}(\alpha^2 L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.35×10^{-3}	$\xrightarrow{1/2}$	0.2×10^{-3}
$\mathcal{O}(\alpha^3 L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.45×10^{-3}	$\xrightarrow{1/8}$	0.05×10^{-3}

Table 1: **The canonical coefficients indicating the generic magnitude of various leading and sub-leading contributions up to third-order. The big-log L is calculated for $\theta = 25\ mrad$.**

now the same thing, in order to avoid double counting, we should take instead of the sum the maximum of the two. This gives us roughly a 0.15% estimate of the *total bremsstrahlung uncertainty* in BHLUMI.4, i.e. the corresponding physical and technical precision. This is a net improvement over the published 0.24%! In order to improve on the above we have to have a better, separate, estimate of the technical precision for the *experimental* trigger below 0.05% and a better estimate (by direct calculation) of the $\mathcal{O}(\alpha^2 L)$ missing contribution. Of course, inclusion of the error from the vacuum polarization, light fermion pairs, etc., will increase the error but not too much. This more detailed analysis will be presented elsewhere.

The central question of the *physical precision* of the BHLUMI.4 problem is now obviously the following: How big is the missing second-order subleading $\mathcal{O}(\alpha^2 L)$ correction. The best would be to calculate this object *directly* and there are attempts in this direction of the present group and others[11]. For the moment let us gather all *indirect* information on this subject. On the one hand the previous[1] estimate of 0.1% is in good agreement with the 0.15% variation in Fig. 6. On the other hand, there is also some indication that the actual value may be smaller. This comes mainly from the analytical inspection of the difference BHLUMI.4 $-$ (OLDBIS + LUMLOG) for the academic trigger, see also in Fig. 5b, and confirmed to some extent by Fig. 7c for the real experimental trigger.

The closer look into $\mathcal{O}(\alpha^2 L)$ terms in the analytical formula for BHLUMI.4 $-$ (OLDBIS + LUMLOG) reveals that almost all such terms are numerically unimportant because the value of the coefficient $(\alpha/\pi)^2 4L = 0.035\%$ is small. All terms which do matter numericaly have special resons for this. They are bigger because they include *enhanecment factors* which can be understood and traced back to some peculiarities of the calculation. In the case of Fig. 5b all these enhancement factors reflect either a lack of exponentiation in OLDBIS or a zero-transverse-momentum

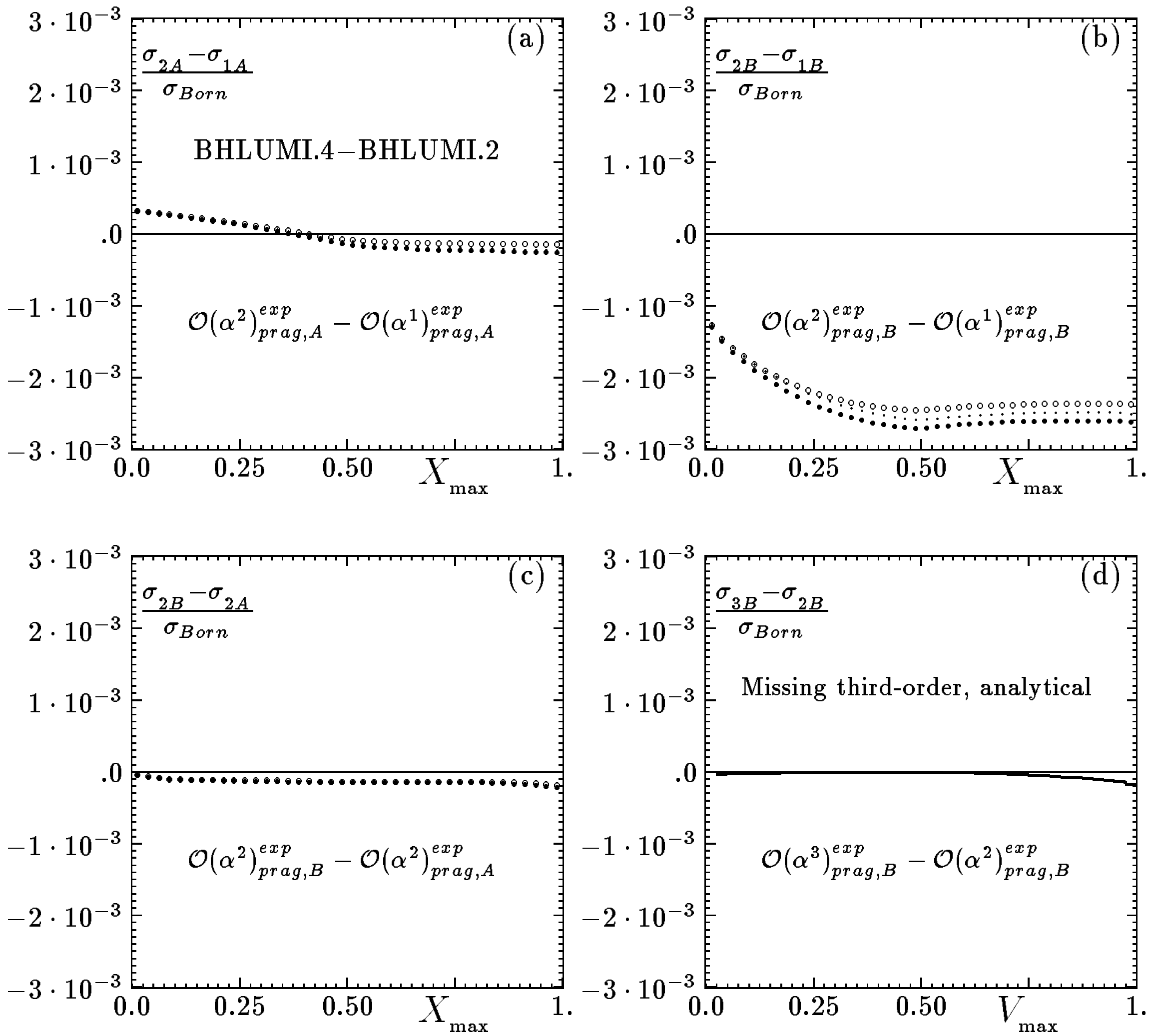


Figure 7: Plots (a)-(c) show Monte Carlo results for SICAL/ALEPH detector and (d) shows analytical result for "academic" trigger. All cross sections are divided by Born value and plotted as a function of the energy cut $X_{\max}$ or $V_{\max}$. In (a) we demonstrate "missing second-order" of published BHLUMI.2 as a difference with new version BHLUMI.4. In (b) we plot the same quantity for matrix element type B. Plot (c) demonstrates difference of results with A and B type $\mathcal{O}(\alpha^2)_{prag}$ matrix elements. In (c) we show for "academic" trigger how the cross section would change if the matrix element B was upgraded to $\mathcal{O}(\alpha^3)_{prag}$.

approximation in LUMLOG. (They are typically logarithmss of cut-off parameters and $\zeta_2 = \pi^2/6$ in the virtual corrections.) The fact that the result shown in Fig. 5b is bigger than the canonical 0.035% reflects these artefacts in the OLDBIS+LUMLOG and not that we miss such contributions in BHLUMI.4. The important lesson for future evaluation of $\mathcal{O}(\alpha^2 L)$ contributions is that any such contribution above 0.035% has to be checked and explained separately because it has to have a special reason, the enhancement factor, to exist! We have done the above exercise in a bit more systematic way and, looking into coefficients in the eq. (2), we have obtained all typical "canonical" coefficients for various leading and subleading contributions up to third-order, which are included in Table 1. Of course the coefficients are smaller for calorimetric detection of the final state electrons and we tried also to estimate this effect. Note that our canonical coefficients of Table 1 agree very well for $\mathcal{O}(\alpha^2 L^2)$ with Fig. 7b, for $\mathcal{O}(\alpha^2 L)$ with Fig. 7c, and for $\mathcal{O}(\alpha^3 L^3)$ with Fig. 7d.

5. Summary and outlook

The status of our work on the QED corrections for the luminosity cross section can be summarized as follows:

- The $\mathcal{O}(\alpha^2)_{prag}$ exponentiated matrix element in the new version of BHLUMI.4 is implemented and we have a lot of high-precision (technical precision: 3×10^{-4}) evidence that it was done correctly.

- A technical precision as high as 3×10^{-4} is established for BHLUMI.4 for the special "academic' type of trigger (cut-off's).

- The OLDBIS/LUMLOG tandem is used to port the above high technical precision to realistic examples of triggers (under development).

- There are indications that the main $\mathcal{O}(\alpha^2 L)$ contribution to physical precision is below 0.1% (even 0.03% is possible) but we have to stick to a conservative estimate of 0.1%, which provides us the new BHLUMI.4 total bremsstrahlung uncertainty of 0.15% for the generic experimental trigger. This is a considerable improvement over the previously published value of 0.24%.

In the future work we plan to extend the technical precision 3×10^{-4} to true realistic experimental triggers, implement $\mathcal{O}(\alpha^2 L)$ part of the matrix element or to provide more solid numerical evaluation/estimate of its magnitude. Realization of the above will definitely allow bringing the total precision below 0.1%, which will match the best experimental errors.

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