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Precision Calculation of the γ–Z Interference Effect in the SLC/LEP Luminosity Process§

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Abstract

We calculate the $\mathcal{O}(\alpha)$ YFS exponentiated contribution of the Z boson, δ_Z, to the SLC/LEP luminosity process of low-angle Bhabha scattering. We realize our results via Monte Carlo methods and discuss their role in the theoretical uncertainty of the luminosity measurement. In the angular range 1.5°–3° we estimate the total precision of our results for δ_Z as 0.015%; in the angular range 3°–6° we estimate the total precision of our results for δ_Z as 0.09%. This represents a clear improvement over what is currently available in the literature, with which we make many cross-checks.

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Recently, the LEP Collaborations all made significant progress in reducing the pure experimental error in their luminosity measurements and some of them [1, 2] have already reached a precision better than 0.1% in this error. The theoretical predictions for these measurements must therefore be improved to the same below 0.1% regime in order that they do not unnecessarily impede the respective high precision tests of the Standard Model in Z physics at SLC/LEP.

In this paper, we present calculations which allow us to reduce the size of the error in one important contribution, that of the Z boson, to the respective luminosity process as it is currently calculated with the YFS exponentiated [3] Monte Carlo event generator BHLUMI 4.02 [4] of two of the present authors (S. J. and B.F.L. W.). The Z boson contribution in the low-angle Bhabha scattering process used for luminosity measurements at SLC/LEP is mainly the effect of the interference between the t-channel γ_t and the s-channel Z_s denoted as $\gamma_t \otimes Z_s$. This contribution vanishes quickly at low-angles and in the angular range ($3°$–$6°$) of the first generation LEP luminometers it is $\lesssim 1\%$ while in the second generation of LEP luminometers ($1.5°$–$3°$) it is at most $\lesssim 0.2\%$. Strictly speaking this contribution in the luminosity cross section should be absent and if the precision of LEP luminometers was as originally planned around 1% then this would be effectively true. With the present precision of less than 0.1% the Z contribution must be discussed very carefully and removed from the luminosity cross section[1]. How was the Z contribution treated in the past? In the BHLUMI Monte Carlo program [5, 6], which was used since 1992 by the LEP collaborations, this contribution was included in the Born approximation only. As was pointed out [7] this was not enough because QED corrections to this contribution are sizeable (up to 50%). Subsequently Beenakker and Pietrzyk proposed in refs. [8, 9] to derive the Z contribution, hereafter referred to as δ_Z, from the Monte Carlo program BABAMC [10] of Berends, Kleiss and Hollik, where it is calculated to $\mathcal{O}(\alpha)$, a net improvement over the Born level. The semi-analytical program ALIBABA [11, 12] of Beenakker, Berends and van der Marck was used to estimate the technical and physical precision of the $\mathcal{O}(\alpha)$ result for δ_Z for simplified cuts; this estimate was extrapolated to the realistic experimental cuts in a conservative manner. The δ_Z contribution obtained in this way was then (typically) combined with the BHLUMI Monte Carlo result[2]. The above represents the current state of the art in calculating this contribution, see also ref. [1]. In this way, the current error on the δ_Z contribution for the new LEP luminometer angular range, $1.5°$–$3°$, is determined by Beenakker and Pietrzyk to be $\sim 0.042\%$, for example.

In the present work we aim at improving on two aspects of the method of calculation of δ_Z proposed in refs. [8, 9]. We want to provide a more precise calculation of δ_Z and we want to release BHLUMI users from the need of using external Monte Carlo calculations for δ_Z correction. The latter improvement is of great practical importance because the use of several Monte Carlo programs is cumbersome and may lead to additional technical

[1]Note that removing the Z contribution simply by putting luminometers below $1°$ is difficult (without compromising on the experimental error) because of the finite size of the beam pipe.

[2]Note that the OPAL Collaboration have used the Monte Carlo of ref. [13] instead of BABAMC to calculate the $\mathcal{O}(\alpha)$ value of δ_Z.

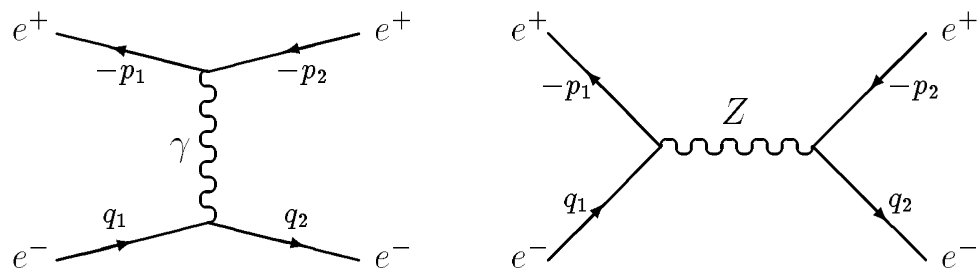

Figure 1: low-angle Bhabha scattering: Born contribution to the $\gamma_t \otimes Z_s$ interference effect.

errors.

Our work is organized as follows: (a) we shall first implement an $\mathcal{O}(\alpha)$ (no exponentiation) version of the Z boson contribution in the BHLUMI program, reproducing precisely the results of refs. [8, 9] in the entire energy regime discussed therein from 89.661 GeV to 92.661 GeV; (b) in the next step we construct the prediction for δ_Z from the $\mathcal{O}(\alpha)$ YFS-exponentiated BHLUMI program. We compare our results with those presented in refs. [8, 9] for all aspects of the corrections to δ_Z, both at $\mathcal{O}(\alpha)$ and higher orders. In this way, our final result for δ_Z is made available as version 4.02 of BHLUMI so that it may be applied to arbitrary detector cuts; (c) finally, we discuss our error estimates and summarize our results. A result of our checks and comparisons with refs. [8, 9] is then a new solid estimate of the technical/physical precision of our new MC realization of δ_Z.

In the following we shall present analytical differential distributions relevant for the Z boson effect δ_Z in the differential cross section for $e^+e^- \longrightarrow e^+e^-$ process at low-angles in the SLC/LEP luminosity regime in the $\mathcal{O}(\alpha)$ and YFS-exponentiated $\mathcal{O}(\alpha)$. We start with the $\mathcal{O}(\alpha)$ case, no exponentiation.

The relevant kinematics is illustrated in fig. 1, where the Born diagrams of interest to us are shown. We follow the work of Berends and Komen in ref. [14] to calculate the $\mathcal{O}(\alpha)$ virtual corrections to the process in fig. 1 and we use the CALKUL methods [15] as formulated by Xu *et al.* [16] to compute the explicit expression for the hard $\mathcal{O}(\alpha)$ bremsstrahlung correction to this process. For the $\mathcal{O}(\alpha)$ virtual corrections, we get in this way the following differential cross section for the $\gamma_t \otimes Z_s$ interference including $\mathcal{O}(\alpha)$ virtual and real soft photon (i.e. for $E_\gamma \le k_m \ll E_{beam}$)

$$\frac{d\sigma^{\gamma_t\otimes Z_s}}{d\Omega} = \frac{d\sigma_0^{\gamma_t\otimes Z_s}}{d\Omega}\left\{1+\left(\frac{\alpha}{\pi}\right)\left((1+2\tilde{u}-2\tilde{v}-2\tilde{w})\left[\ln\frac{s}{4k_m^2}+\frac{\tilde{\Gamma}(s)}{\sqrt{s}-M}\,\phi\right]\right.\right.$$
$$+(1-2\tilde{v})\ln\frac{sR(k_m)}{4k_m^2R(0)}+2(\tilde{u}-\tilde{w})\ln\frac{R(k_m)}{k_m^2}-2\tilde{u}^2+2\tilde{v}^2$$
$$+4\tilde{w}(\tilde{u}-\tilde{v})+3\tilde{v}+3\tilde{w}+Li_2(1-\zeta)-Li_2(\zeta)+\frac{\pi^2}{6}-4$$

$$+\left[\frac{\Im\bar{B}(s)}{\Re\bar{B}(s)}\right]\pi\left[2(\tilde{w}-\tilde{v})+4(\tilde{u}-\tilde{w})\frac{\psi}{\pi}+\frac{3}{2}\right]\Bigg)\Bigg\}, \tag{1}$$

where the lowest-order cross section for $\gamma_t \otimes Z_s$ interference is

$$\frac{d\sigma_0^{\gamma_t\otimes Z_s}}{d\Omega}=\frac{\alpha(t)^2}{s\zeta^2}\left[-\zeta(1-\zeta)^2\right]\left[\Re\bar{B}(s)\right], \qquad \zeta=\frac{|t|}{s}, \tag{2}$$

and

$$\bar{B}(s)=\frac{\alpha(s)}{\alpha(t)}(v^2+a^2)\frac{s}{s-M^2+iM\tilde{\Gamma}(s)}, \tag{3}$$

$$R(k)=(\sqrt{s}-k-M)^2+\frac{1}{4}\tilde{\Gamma}(s)^2, \tag{4}$$

$$\phi=\arctan\left(\frac{-2(\sqrt{s}-M)+2k_m}{\tilde{\Gamma}(s)}\right)-\arctan\left(\frac{-2(\sqrt{s}-M)}{\tilde{\Gamma}(s)}\right), \tag{5}$$

$$\psi=\arccos\frac{M^2-s}{\sqrt{(s-M^2)^2+[M\tilde{\Gamma}(s)]^2}}, \quad \psi\in[0,\pi]. \tag{6}$$

Here, we also defined $\tilde{u}=(1/2)\ln(|u|/m_e^2)$, $\tilde{v}=(1/2)\ln(s/m_e^2)$, $\tilde{w}=(1/2)\ln(|t|/m_e^2)$, where s is the CMS energy squared, t is the 4-momentum transfer squared, and $u=-s-t$. Furthermore, $\tilde{\Gamma}(s)=s\Gamma/M^2$, where Γ and M are the Z width and the rest mass, respectively, and the electroweak coupling constants are $v=a(1-4\sin^2\theta_W)$ for $a=-1/(4\sin\theta_W\cos\theta_W)$, where θ_W is the electroweak mixing angle.

Why did we use a formula of ref. [14] which is based on the narrow resonance approximation, instead of using exact results of ref. [17]? Simply because the presented formula is much simpler and also, for our application, it provides the same result, up to 0.5×10^{-4} of the Born cross section, as the exact one. In order to reach this level of precision we had to improve the result of ref. [14] in the limit $k_m \ll \Gamma$. This can be seen in the particular case at the Z position where our formula reads

$$\frac{d\sigma^{\gamma_t\otimes Z_s}}{d\Omega}(\sqrt{s}=M)=\frac{\alpha(t)^2}{s\zeta^2}\frac{\alpha(s)}{\alpha(t)}\left[-\zeta(1-\zeta)^2\right]\left(\frac{\alpha}{\pi}\right)(v^2+a^2)\frac{M}{\tilde{\Gamma}(s)} \tag{7}$$
$$\times\left\{-2(2\tilde{v}+2\tilde{w}-2\tilde{u}-1)\arctan\left(\frac{2k_m}{\tilde{\Gamma}(s)}\right)-\pi\left[2(\tilde{u}-\tilde{v})+\frac{3}{2}\right]\right\},$$

while the original formula did not reproduce correctly either the first term in the curly brackets $\propto \alpha\arctan[2k_m/\tilde{\Gamma}(s)]$ or the second one $\propto \alpha\pi$. Note also that, as seen explicitly in eqs. (1) and (7), and as expected from the Yennie-Frautschi-Suura work [3], the soft divergence, i.e. the term proportional to $\ln k_m$, is governed by the "big logarithm" $\ln(Q^2/m_e^2)$ with $Q^2=|t|$ and not $Q^2=s$. The above is true for $k_m \ll \Gamma$ and is not necessarily in contradiction with ref. [8] where $Q^2=s$ was chosen. In the case $k_m > \Gamma$

the Z resonance propagator may inhibit the destructive interference which is responsible for the $Q^2 = |t|$ scale in soft emission at low-angles.

Having defined the $\mathcal{O}(\alpha)$ (soft + virtual) corrections to δ_Z, let us work out the $\mathcal{O}(\alpha)$ correction to the non-infrared residual $\bar{\beta}_0$ in the Yennie-Frautschi-Suura (YFS) scheme [3]. The relevant YFS-exponentiated total cross section has generally the following form:

$$\sigma_{YFS} = \exp\left[2\alpha\Re B + 2\alpha\tilde{B}(K_{max})\right] \sum_{n=0}^{\infty} \int \frac{d^4y}{(2\pi)^4}\, e^{iy(p_1+q_1-p_2-q_2-\sum_{j=1}^{n} k_j)+D} \times \prod_{j=1}^{n} \int \frac{d^3k_j}{k_j}\, \bar{\beta}_n(k_1,\dots,k_n)\, \frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}, \tag{8}$$

$$D = \int \frac{d^3k}{k}\, \tilde{S}(k)\left(e^{-iky} - \Theta(K_{max} - k^0)\right),$$

where the YFS infrared functions $\tilde{S}(k)$, B and $\tilde{B}$ may be found explicitly in refs. [3, 5], for example; K_{max} is a dummy infrared separation parameter of which (8) is independent and $\bar{\beta}_n$ are the YFS n-real-photon residuals which have neither real nor virtual infrared singularities. The $\mathcal{O}(\alpha)$ interference $\bar{\beta}^{(1)}_{\gamma_t\otimes Z_s,0}$ is defined as follows, in the YFS framework, in the normalization of ref. [5]

$$\frac{1}{2}\bar{\beta}^{(1)}_{\gamma_t\otimes Z_s,0} = \frac{d\sigma^{\gamma_t\otimes Z_s}}{d\Omega} - 2\alpha\left[\Re B + \tilde{B}(k_m)\right]\frac{d\sigma_0^{\gamma_t\otimes Z_s}}{d\Omega}. \tag{9}$$

Substituting the expression of eqs. (1) and (2) into eq. (9) we get

$$\bar{\beta}^{(1)}_{\gamma_t\otimes Z_s,0} = \frac{2\alpha(t)^2}{s}\,\frac{(t_p+t_q)(u^2+u_1^2)}{4t_pt_q}\,\frac{1}{2}\left[Y_{\gamma_t\otimes Z_s}(s) + Y_{\gamma_t\otimes Z_s}(s_1)\right], \tag{10}$$

$$\begin{aligned} Y_{\gamma_t\otimes Z_s}(s) = \left[\frac{\Re\bar{B}(s)}{s}\right]\Bigg\{1 + \left(\frac{\alpha}{\pi}\right)\Bigg(& 2(\tilde{w}-\tilde{u})\ln\frac{s}{4(\sqrt{s}-M)^2+\tilde{\Gamma}(s)^2} + 2(\tilde{v}^2-\tilde{u}^2) \\ &+ 4\tilde{w}(\tilde{u}-\tilde{v}) + 2\tilde{u} + \tilde{v} + \tilde{w} + Li_2(1-\zeta) - Li_2(\zeta) - \frac{\pi^2}{2} - 2 \\ &+ \left[\frac{\Im\bar{B}(s)}{\Re\bar{B}(s)}\right]\pi\left[2(\tilde{w}-\tilde{v}) + 4(\tilde{u}-\tilde{w})\frac{\psi}{\pi} + \frac{3}{2}\right]\Bigg)\Bigg\}, \end{aligned} \tag{11}$$

where $t_p = (p_1-p_2)^2$, $t_q = (q_1-q_2)^2$, $u = (p_1-q_2)^2$, $u_1 = (q_1-p_2)^2$. The second term in the square brackets of eq. (10) takes into account the CMS energy shift of the incoming leptons due to initial state photon radiation: $s \to s_1$, where $s = (p_1+q_1)^2$ and $s_1 = (p_2+q_2)^2$. Note that the YFS scheme leaves us the freedom to include or not the above CMS energy shift in $\bar{\beta}_{\gamma_t\otimes Z_s,0}$. Our particular choice minimizes higher-order effects.

For comparison with refs. [8, 9] and for completeness, we have also included the lowest order s-channel $Z_s\otimes Z_s$ contribution

$$\bar{\beta}^{(0)}_{Z_s\otimes Z_s,0} = \frac{2\alpha(t)^2}{s}\,\frac{u^2+u_1^2+t_p^2+t_q^2}{4}\,\frac{1}{2}\left[\frac{|\bar{B}(s)|^2}{s^2} + \frac{|\bar{B}(s_1)|^2}{s_1^2}\right]. \tag{12}$$

We calculate the $\mathcal{O}(\alpha)$ hard bremsstrahlung correction to δ_Z using the methods of Xu *et al.* [16], and we find that the corresponding complete gauge-invariant contribution to the one real photon differential cross section is

$$D_{\gamma_t \otimes Z_s,1} = \frac{2\alpha(t)^2}{4s} \frac{\alpha}{4\pi^2} (A_0 + A_1), \tag{13}$$

where $A_1 = A_1^U + A_1^L$ takes into account the $s \to s_1$ shift due to initial-state radiation, and $A_0 = A_0^U + A_0^L$ is the no-shift part. The contributions A_i^U represent the emission from the upper (positron) line and A_i^L are from the lower (electron) line. All four components read as follows:

$$A_0^U = \frac{d(s)}{t_q}\Bigg[F(s)\Bigg(-\frac{t_p}{(p_1k)(p_2k)} + \frac{s_1}{(p_2k)(q_2k)} + \frac{u}{(p_1k)(q_2k)} + \frac{4}{p_2k} - \frac{4u^2}{u^2+u_1^2}\frac{m_e^2}{(p_2k)^2}\Bigg) - 2G(s)\,\frac{\epsilon_{\alpha\beta\gamma\delta}p_1^\alpha q_1^\beta p_2^\gamma k^\delta}{(p_1k)(p_2k)(q_2k)}\Bigg], \tag{14}$$

$$A_1^U = \frac{d(s_1)}{t_q}\Bigg[F(s_1)\Bigg(-\frac{t_p}{(p_1k)(p_2k)} + \frac{s}{(p_1k)(q_1k)} + \frac{u_1}{(q_1k)(p_2k)} - \frac{4}{p_1k} - \frac{4u_1^2}{u^2+u_1^2}\frac{m_e^2}{(p_1k)^2}\Bigg) + 2G(s_1)\,\frac{\epsilon_{\alpha\beta\gamma\delta}p_1^\alpha q_1^\beta p_2^\gamma k^\delta}{(p_1k)(p_2k)(q_1k)}\Bigg], \tag{15}$$

$$A_0^L = \frac{d(s)}{t_p}\Bigg[F(s)\Bigg(-\frac{t_q}{(q_1k)(q_2k)} + \frac{s_1}{(p_2k)(q_2k)} + \frac{u_1}{(q_1k)(p_2k)} + \frac{4}{q_2k} - \frac{4u_1^2}{u^2+u_1^2}\frac{m_e^2}{(q_2k)^2}\Bigg) - 2G(s)\,\frac{\epsilon_{\alpha\beta\gamma\delta}p_1^\alpha q_1^\beta p_2^\gamma k^\delta}{(q_1k)(q_2k)(p_2k)}\Bigg], \tag{16}$$

$$A_1^L = \frac{d(s_1)}{t_p}\Bigg[F(s_1)\Bigg(-\frac{t_q}{(q_1k)(q_2k)} + \frac{s}{(p_1k)(q_1k)} + \frac{u}{(p_1k)(q_2k)} - \frac{4}{q_1k} - \frac{4u^2}{u^2+u_1^2}\frac{m_e^2}{(q_1k)^2}\Bigg) + 2G(s_1)\,\frac{\epsilon_{\alpha\beta\gamma\delta}p_1^\alpha q_1^\beta p_2^\gamma k^\delta}{(q_1k)(q_2k)(p_1k)}\Bigg], \tag{17}$$

where

$$d(x) = \frac{\alpha(x)}{\alpha(t)}\left[(x-M^2)^2 + (M\tilde{\Gamma}(x))^2\right]^{-1}, \tag{18}$$

$$F(x) = (v^2+a^2)(u^2+u_1^2)(x-M^2), \qquad G(x) = 2va(u^2-u_1^2)M\tilde{\Gamma}(x). \tag{19}$$

In the YFS-exponentiated case we have to provide non-infrared one-real-photon components of the $\bar{\beta}_1$-type. Again we split them according to

$$\bar{\beta}^{(1)}_{\gamma_t\otimes Z_s,1} = \bar{\beta}^{(1)}_{\gamma_t\otimes Z_s,1U} + \bar{\beta}^{(1)}_{\gamma_t\otimes Z_s,1L} \tag{20}$$

They are defined, in the YFS scheme, in this special case of the interference contribution, as follows

$$\begin{aligned}
\bar\beta^{(1)}_{\gamma_t\otimes Z_s,1U} &= D^U_{\gamma_t\otimes Z_s,1} - \bar\beta^{(0)}_{\gamma_t\otimes Z_s,0} \\
&\qquad\qquad \times\left(\tilde S(p_1,p_2)+\frac{1}{2}\left[\tilde S(p_1,q_1)+\tilde S(p_2,q_2)-\tilde S(p_1,q_2)-\tilde S(q_1,p_2)\right]\right), \\
\bar\beta^{(1)}_{\gamma_t\otimes Z_s,1L} &= D^L_{\gamma_t\otimes Z_s,1} - \bar\beta^{(0)}_{\gamma_t\otimes Z_s,0} \\
&\qquad\qquad \times\left(\tilde S(q_1,q_2)+\frac{1}{2}\left[\tilde S(p_1,q_1)+\tilde S(p_2,q_2)-\tilde S(p_1,q_2)-\tilde S(q_1,p_2)\right]\right),
\end{aligned} \tag{21}$$

where $\tilde S(p,q) \equiv -(\alpha/4\pi^2)[p/(pk)-q/(qk)]^2$ and $\bar\beta^{(0)}_{\gamma_t\otimes Z_s,0}$ can be obtained from eq. (10) simply by dropping the $\mathcal{O}(\alpha)$ correction term. Note that in eq. (21) and in previous $\mathcal{O}(\alpha)$ formulas the so-called "up down" QED interferences[3] are included while in the rest of the BHLUMI matrix element they are routinely neglected (they are at low-angles totally negligible, see ref. [18]). This should be understood as follows: our $\bar\beta^{(1)}_{\gamma_t\otimes Z_s,1}$ is in fact calculated *in the presence* of all up down interferences, as in eq. (8), and up down interferences are dropped afterwards in all terms in the class $\gamma_t\otimes\gamma_t$, but not in contributions of the type $\gamma_t\otimes Z_s$. Such a procedure does not violate cancelation of infrared divergences because it is done for the infrared-finite objects, i.e. for $\bar\beta_i$'s of the YFS scheme, and is also gauge-invariant because all building blocks in eq. (21) (notably $D_{\gamma_t\otimes Z_s,1}$) are gauge-invariant. Finally, let us remember that, since formula (8) is a rearrangement of the exact Feynman-Schwinger-Tomonaga perturbation series for the low-angle Bhabha process, it is therefore guaranteed that if we truncate our $\mathcal{O}(\alpha)$ YFS-exponentiated differential distributions relevant to δ_Z back to ordinary $\mathcal{O}(\alpha)$ then we recover our original $\mathcal{O}(\alpha)$ differential distributions.

In the following we shall now check carefully that the distributions defined in eqs. (1)–(21) as implemented in BHLUMI 4.02 agree very well with the wealth of numerical results presented in refs. [8, 9], which were obtained using the BABAMC Monte Carlo [10] and the ALIBABA [11, 12] semi-analytical code. Note that our expressions in eqs. (1)–(21) are much simpler and more compact than those in BABAMC and ALIBABA because we restrict ourself to Z exchange contributions in the form relevant to low-angle Bhabha scattering. In order to find a numerical contact with the results of refs. [8, 9] we had to use the same input parameters (obsolete values of the Z mass and width) and in some cases rearrange the vacuum polarization and Z self-energy corrections in the additive form (instead of multiplicative). Furthermore, in refs. [8, 9] practically all results are presented for the acceptance which requires the outgoing electron and positron to stay within a certain angular range $(\theta_{min},\theta_{max})$ without any cut on the photon momentum. This acceptance is rather unrealistic because the real LEP experiments do not distinguish photons and electrons (calorimetric detection) and also some kind of minimum of the detected energy is imposed. Such a non-calorimetric acceptance was used in refs. [8, 9] because ALIBABA is not able to provide results for an acceptance closer to the real

[3]Contributions from diagrams in which an additional photon line connects electron and positron lines.

$\sqrt{s}$ [GeV]	Z-Born+$\mathcal{O}(\alpha)$			QED $\mathcal{O}(\alpha)$		Remaining $\mathcal{O}(\alpha)$	
	BHLUMI	BABAMC	ALIBABA	BHLUMI	BABAMC	BHLUMI	BABAMC
89.661	+0.805	+0.774	+0.781	−0.242	−0.248	+0.118	+0.100
90.036	+0.838	+0.816	+0.811	−0.237	−0.236	+0.126	+0.110
90.411	+0.795	+0.774	+0.771	−0.176	−0.197	+0.116	+0.121
90.786	+0.603	+0.588	+0.592	−0.018	−0.030	+0.071	+0.071
91.161	+0.247	+0.243	+0.245	+0.225	+0.212	+0.000	+0.009
91.536	−0.149	−0.146	−0.142	+0.433	+0.421	−0.053	−0.044
91.911	−0.436	−0.438	−0.426	+0.510	+0.498	−0.075	−0.072
92.286	−0.587	−0.590	−0.571	+0.485	+0.455	−0.078	−0.059
92.661	−0.643	−0.637	−0.625	+0.422	+0.406	−0.075	−0.060

Table 1: First-order (no exponentiation) contributions/corrections to δ_Z (see eq. (22) for definition of δ_Z) for $M_Z = 91.161\,\mathrm{GeV}$ and $\Gamma_Z = 2.487\,\mathrm{GeV}$ and for the non-calorimetric acceptance of ref. [8] in the LCAL angular range: $3.3^\circ \leq \theta \leq 6.3^\circ$. Statistical errors of the BHLUMI and BABAMC results are $\leq 0.001\%$ and $\leq 0.013\%$, respectively. See text for precise definitions of the presented contributions.

experiment. The angular acceptance in ref. [8] was that of the ALEPH LCAL detector, i.e. $3.3^\circ \leq \theta \leq 6.3^\circ$, and in ref. [9] it was that of the new ALEPH SICAL and OPAL SiW detectors at angles below 3°. For obvious reasons we shall use, in the following comparisons, exactly the same type of acceptances as in refs. [8, 9].

In the following comparisons with refs. [8, 9] we look into various contributions to the quantity

$$\delta_Z = \frac{\sigma_Z}{\sigma_{Born}} \tag{22}$$

in which σ_Z is the cross section involving exchange of the s-channel Z_s and σ_{Born} is the complete Born cross section for the Bhabha process (involving all γ and Z exchanges) as defined in table 1 in ref. [8] or table 1 in ref. [9].

In table 1, for the LCAL angular range, we show a comparison of our $\mathcal{O}(\alpha)$ results for δ_Z as calculated by the $\mathcal{O}(\alpha)$ version of BHLUMI 4.02 without exponentiation and as calculated with BABAMC and ALIBABA from ref. [8]. More precisely we include here for BABAMC the results from table 6 in ref. [8] and for ALIBABA the results from table 8 in ref. [8]. We have run BABAMC in order to reproduce these results once again. In the case of BHLUMI 4.02 we had to rearrange the vacuum polarization and Z self-energy accordingly to get the presented good agreement. In the table 1 we keep the same terminology as in ref. [8]. Let us recall the definitions of the various contributions. The contribution "Z-Born+$\mathcal{O}(\alpha)$" represents the entire Z contribution as calculated in the $\mathcal{O}(\alpha)$. This is split into the lowest-order "Z-Born" and the "$\mathcal{O}(\alpha)$ correction". The "QED $\mathcal{O}(\alpha)$" represents the pure bremsstrahlung (photonic) part of the "$\mathcal{O}(\alpha)$ correction" and the "remaining $\mathcal{O}(\alpha)$" denotes the rest of the "$\mathcal{O}(\alpha)$ correction", i.e. all kinds of vacuum polarizations and self-energies, etc. As we see the $\mathcal{O}(\alpha)$ BHLUMI 4.02 is within the 2σ

statistical errors of the BABAMC results and it agrees with ALIBABA to within 0.03% throughout the entire energy range.

$\sqrt{s}$ [GeV]	Z-Born+$\mathcal{O}(\alpha)$+h.o.		QED correction		Remaining correction	
	BHLUMI	ALIBABA	BHLUMI	ALIBABA	BHLUMI	ALIBABA
89.661	+0.794	+0.778	−0.220	−0.248	+0.085	+0.104
90.036	+0.816	+0.799	−0.221	−0.251	+0.088	+0.108
90.411	+0.754	+0.747	−0.181	−0.200	+0.080	+0.098
90.786	+0.538	+0.545	−0.066	−0.066	+0.054	+0.064
91.161	+0.158	+0.187	+0.120	+0.148	+0.016	+0.017
91.536	−0.247	−0.206	+0.295	+0.335	−0.014	−0.018
91.911	−0.521	−0.479	+0.379	+0.418	−0.029	−0.033
92.286	−0.647	−0.609	+0.383	+0.416	−0.035	−0.039
92.661	−0.678	−0.650	+0.349	+0.374	−0.037	−0.041

Table 2: Higher-order (beyond $\mathcal{O}(\alpha^1)$) contributions/corrections from ALIBABA and the exponentiated BHLUMI to δ_Z (see eq. (22) for a definition of δ_Z) for $M_Z = 91.161\,\mathrm{GeV}$ and $\Gamma_Z = 2.487\,\mathrm{GeV}$ and for the non-calorimetric acceptance of ref. [8] in the LCAL angular range: $3.3° \le \theta \le 6.3°$. Statistical errors of the BHLUMI results are $\le 0.001\%$. See text for definitions of contributions/corrections.

In table 2 we compare δ_Z calculated using BHLUMI 4.02 and ALIBABA beyond $\mathcal{O}(\alpha)$ for a non-calorimetric acceptance in the LCAL angular range, as in table 1. The results from BHLUMI are obtained using the exponentiated distributions for δ_Z presented in this work. The YFS exponentiation provides us with a substantial part of $\mathcal{O}(\alpha^2)$ correction to δ_Z but not all of it[4]. Note that in all our comparisons with BABAMC and ALIBABA, the $\mathcal{O}(\alpha)$ BHLUMI results have the additive vacuum polarization for QED and the additive Z self energy whereas for the exponentiated $\mathcal{O}(\alpha)$ BHLUMI results the vacuum polarization for QED and the Z self energy are Dyson summed into multiplicative corrections. The semi-analytical ALIBABA program includes $\mathcal{O}(\alpha^2)$ LL corrections. In table 2 the results of ALIBABA are taken from table 7 in ref. [8]. In the table 2 we again follow the terminology of ref. [8] in naming the contributions to δ_Z, "Z-Born+$\mathcal{O}(\alpha)$+h.o." denotes total result for δ_Z including the lowest order, "QED correction" denotes the pure bremsstrahlung part of the "total correction" to the lowest order and the "remaining correction" is the rest of the "total correction". As we see in table 2 the BHLUMI result agrees with ALIBABA for the full correction to within 0.042% throughout the entire energy range shown, and we see that the "remaining corrections" are within 0.02% for the two calculations; indeed, in the region where the total predictions differ most, the "remaining corrections" are within 0.004% of one another. This shows that most of the 0.042% difference at 91.911 GeV is due to the different treatments of the pure bremsstrahlung (photonic) higher-order QED corrections. The above comparison shows

[4]In the case of the bulk cross section $\gamma_t \otimes \gamma_t$ it was shown in ref. [19] that YFS exponentiation may add practically all of $\mathcal{O}(\alpha^2)$ leading logarithmic (LL) corrections.

(for this particular non-calorimetric trigger) that both programs have technical precision and $\mathcal{O}(\alpha^2)$ LL corrections under control down to the 0.04% level! This is a definite progress over 0.06% quoted typically[5] for similar comparisons in ref. [8], and much better than 0.12% quoted as the total precision of the BHLUMI+BABAMC recipe in ref [8].

$\sqrt{s}$ [GeV]	QED $\mathcal{O}(\alpha)$			Remaining $\mathcal{O}(\alpha)$		
	BHLUMI	BABAMC	ALIBABA	BHLUMI	BABAMC	ALIBABA
89.661	−0.057	−0.059	−0.058	+0.013	+0.013	+0.012
90.036	−0.058	−0.059	−0.059	+0.010	+0.002	+0.009
90.411	−0.047	−0.042	−0.048	+0.004	−0.015	+0.004
90.786	−0.010	−0.002	−0.012	−0.002	−0.021	+0.000
91.161	+0.052	+0.054	+0.049	+0.000	−0.012	+0.003
91.536	+0.104	+0.100	+0.101	+0.006	−0.008	+0.010
91.911	+0.121	+0.129	+0.117	+0.006	−0.013	+0.010
92.286	+0.112	+0.121	+0.109	+0.001	−0.020	+0.006
92.661	+0.095	+0.088	+0.092	−0.003	−0.022	+0.001

Table 3: First-order (no exponentiation) contributions/corrections to δ_Z (see eq. (22) for a definition of δ_Z) for $M_Z = 91.161\,\mathrm{GeV}$ and $\Gamma_Z = 2.3098\,\mathrm{GeV}$ and for the non-calorimetric acceptance of ref. [9] in the ALEPH SICAL and OPAL SiW detectors with the asymmetric angular range (below 3°). Statistical errors of the BHLUMI results are $\leq 0.001\%$. The contributions/corrections are defined as in table 1.

In table 3, we show predictions analogous to those in table 1, for the same non-calorimetric acceptance, but with the asymmetric angular range corresponding to the ALEPH SICAL and OPAL SiW detectors. As in ref. [9] we choose for asymmetric angular cuts $1.61^\circ < \theta_1 < 2.8^\circ$ and $1.5^\circ < \theta_2 < 3.15^\circ$ and we do not have any cut on the photon momentum. We also substitute $\Gamma_Z = 2.3098\,\mathrm{GeV}$ to facilitate the comparison with ref. [9]. The results for BABAMC and ALIBABA come from table 3 in ref. [9]. We were able to reproduce the BABAMC results with our own high-statistics Monte Carlo runs. We see agreement between BHLUMI at $\mathcal{O}(\alpha)$ QED and BABAMC at the 0.01% level, and between BHLUMI and ALIBABA at $\mathcal{O}(\alpha)$ QED at the 0.004% level. Even for the "remaining $\mathcal{O}(\alpha)$ corrections", the results of BHLUMI at $\mathcal{O}(\alpha)$ and ALIBABA at $\mathcal{O}(\alpha)$ agree at the 0.005% level, whereas for BABAMC the analogous difference in the "remaining correction" reaches 0.021%.

In table 4, we show the "Z-Born+$\mathcal{O}(\alpha)$" and "Z-Born+$\mathcal{O}(\alpha)$+h.o." predictions (see tables 1 and 2 for their definitions) for the same non-calorimetric type of acceptance with the angular range of SICAL and SiW detectors as in the previous table 3. Again we substitute $\Gamma_Z = 2.487\,\mathrm{GeV}$. The results for ALIBABA come from table 5 in ref. [9]. For BABAMC we include results of our own high statistics Monte Carlo run and in brackets we quote the corresponding lower statistics results of ref. [9]. As we see the

[5] In ref. [8] this precision was deduced from comparison of ALIBABA with the results of the approximate formulae of eqs. (5) and (6) in ref. [8].

$\sqrt{s}$ [GeV]	Z-Born + $\mathcal{O}(\alpha)$			Z-Born + $\mathcal{O}(\alpha)$ + h.o.	
	BHLUMI	BABAMC	ALIBABA	BHLUMI	ALIBABA
89.661	+0.178	(+0.176) + 0.175	+0.172	+0.175	+0.172
90.036	+0.185	(+0.172) + 0.181	+0.180	+0.180	+0.177
90.411	+0.173	(+0.156) + 0.171	+0.169	+0.165	+0.164
90.786	+0.129	(+0.119) + 0.127	+0.127	+0.116	+0.117
91.161	+0.049	(+0.036) + 0.049	+0.050	+0.032	+0.036
91.536	−0.038	(−0.048) − 0.037	−0.036	−0.057	−0.050
91.911	−0.100	(−0.105) − 0.097	−0.096	−0.116	−0.110
92.286	−0.132	(−0.141) − 0.128	−0.127	−0.143	−0.136
92.661	−0.143	(−0.156) − 0.140	−0.140	−0.150	−0.145

Table 4: The $\mathcal{O}(\alpha^1)$ and beyond contributions/corrections from ALIBABA and BHLUMI to δ_Z (see eq. (22) for a definition of δ_Z) for $M_Z = 91.161\,\mathrm{GeV}$ and $\Gamma_Z = 2.487\,\mathrm{GeV}$ and for the non-calorimetric acceptance of ref. [9] in the SICAL/SiW asymmetric angular range (below 3°). Statistical errors of the BHLUMI results are $\leq 0.001\%$. The results for BABAMC are with the statistical error of about 0.001% and were obtained by us from special high-statistics runs. In parentheses we quote BABAMC results from ref. [9]. The contributions/corrections are defined as in the previous tables.

"Z-Born+$\mathcal{O}(\alpha)$" results for BHLUMI and BABAMC, when the latter is seen for high-statistics, agree at the 0.004% level; for ALIBABA, the respective difference with the BHLUMI "Z-Born+$\mathcal{O}(\alpha)$" results is at or below 0.006%. For the higher-order results, the YFS-exponentiated BHLUMI ones differ from those of ALIBABA by at most 0.007%. These results, when compared with the typical uncertainty 0.015% quoted in ref. [9] give us a good/improved estimate on both the technical and the physical precisions of our δ_Z implementation in BHLUMI and ALIBABA. Strictly speaking, the validity of this result is limited to this particular non-calorimetric acceptance (without energy cut) amenable to the ALIBABA code.

Having shown our comparisons of the new BHLUMI with other generators for various non-calorimetric examples of the acceptance, let us come back to the question of the technical/physical precision of the implementation of δ_Z in BHLUMI in the presence of the realistic experimental cuts. For the experimental acceptance of the SICAL-type luminometer, only a Monte Carlo event generator can provide a prediction for the Z boson contribution (ALIBABA cannot be used any more). Thus, we now turn to a comparison of the event generators BABAMC and BHLUMI at the $\mathcal{O}(\alpha)$. For technical reasons we do it in two steps, first for the real emission part of the matrix element and then for the soft/virtual part. This is done in this way because BABAMC is rather inefficient in calculations of the Z contribution due to prohibitive slowness of the calculation of the hard bremsstrahlung matrix element, which makes it very difficult to reach a low statistical error. In the first step of our $\mathcal{O}(\alpha)$ comparisons between BABAMC and BHLUMI, we keep the same soft/virtual distributions in both programs, that is as in BABAMC. More precisely, inside the $\mathcal{O}(\alpha)$ BHLUMI we replace the distributions given in eq. (1) with

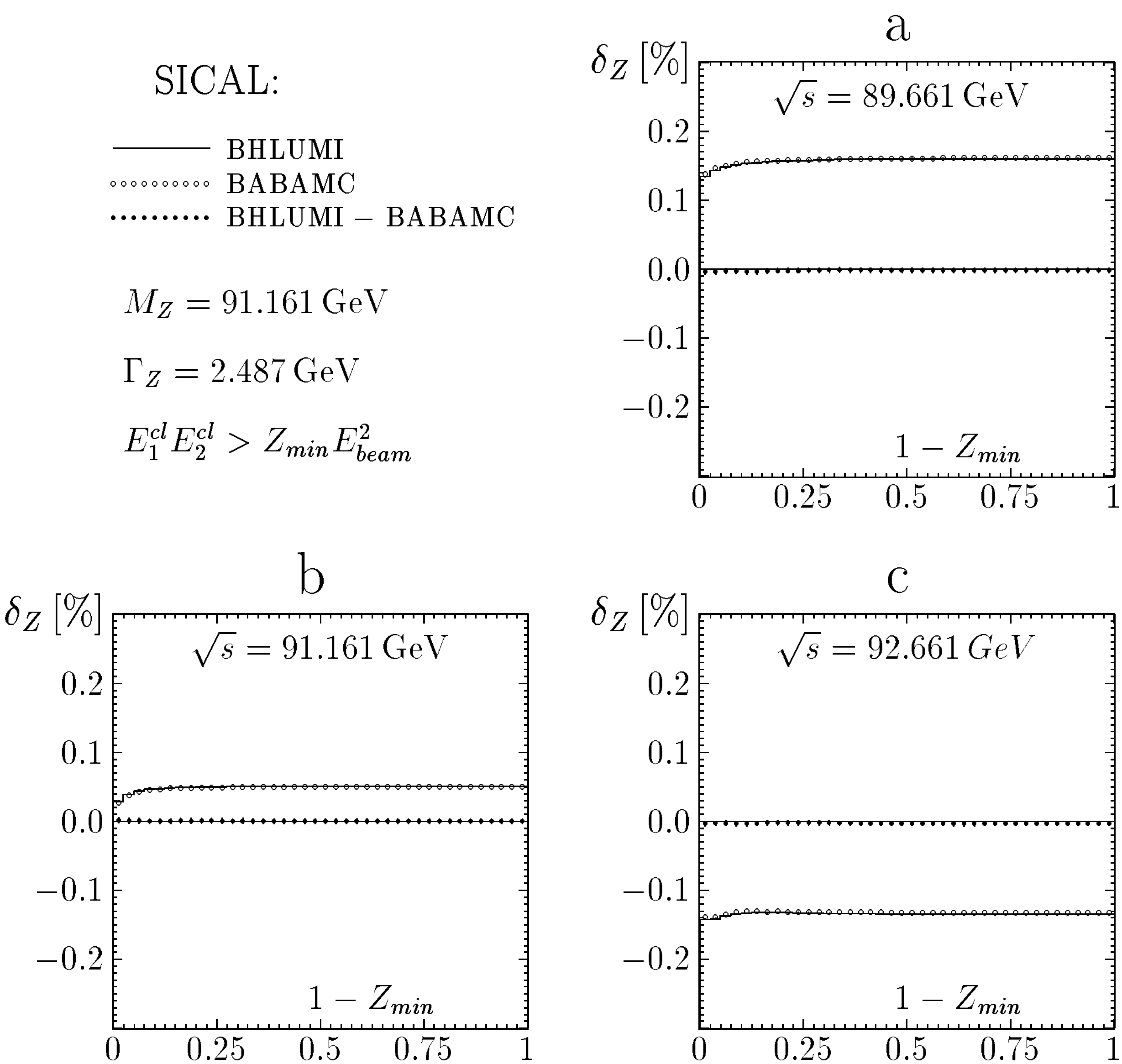

Figure 2: Comparison of the $\mathcal{O}(\alpha)$ hard bremsstrahlung correction to the Z-contribution of BHLUMI and BABAMC for the ALEPH SICAL detector. $\mathcal{O}(\alpha)$ virtual+soft photon corrections in BHLUMI are in this case taken from BABAMC. All results are given as a fraction (in %) of the Born cross section for the pure t-channel photon exchange.

soft/real distributions taken from BABAMC. As for the real emission distributions we keep in BABAMC the original matrix element while in BHLUMI we keep the distributions given in eq. (13). In fig. 2 we prove that this replacement affects the results of BABAMC by less than 0.005% of the Born cross section. The comparison is done for the SICAL acceptance for energies below, at and above the Z peak. For a detailed description of the

SICAL trigger (acceptance) we refer the reader to[6] refs. [1, 19]

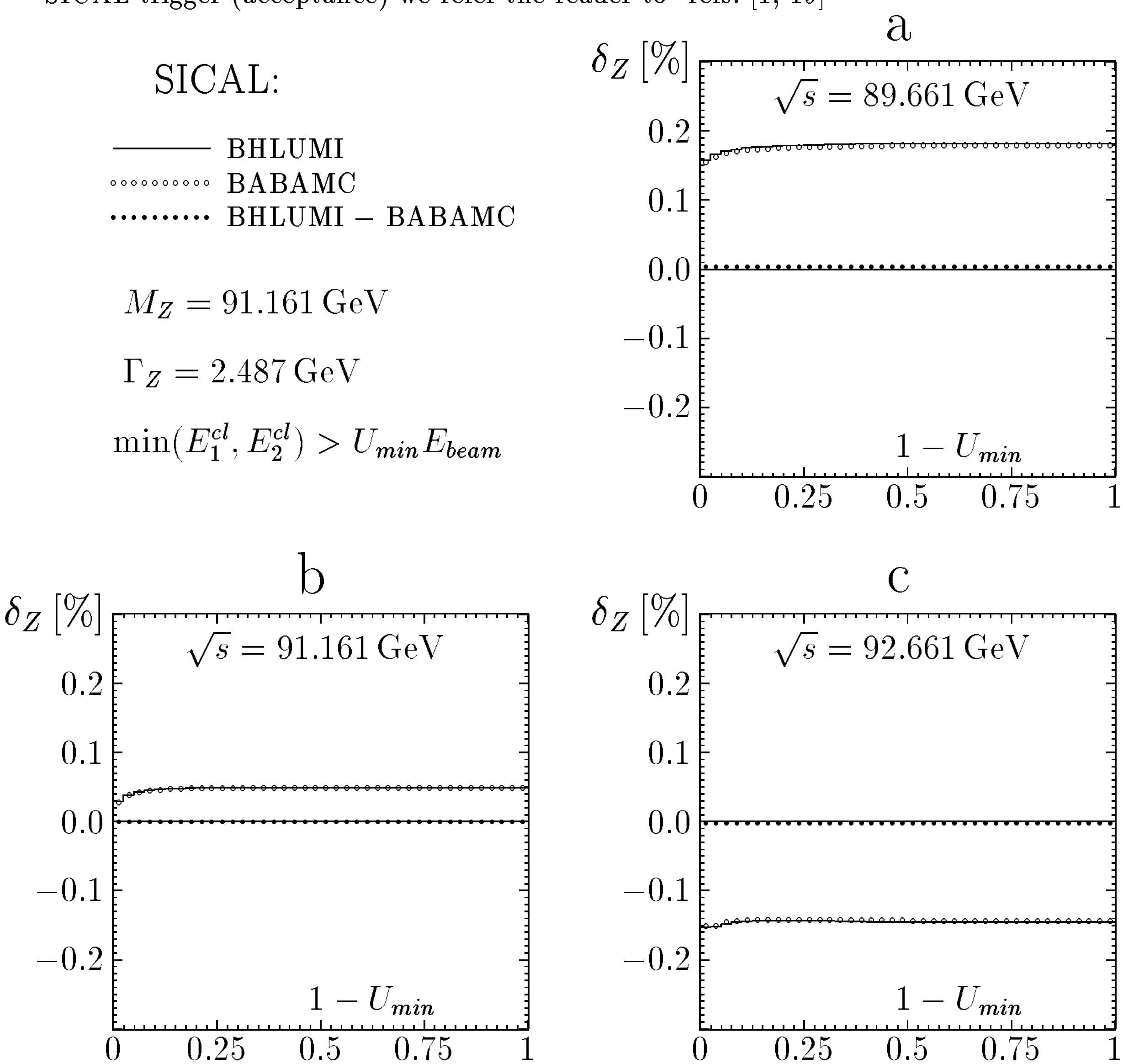

Figure 3: Comparison of the Z-contribution including $\mathcal{O}(\alpha)$ radiative corrections of BHLUMI and BABAMC for the ALEPH SICAL detector. All results are given as a fraction (in %) of the Born cross section for the pure t-channel photon exchange.

In the second step of our $\mathcal{O}(\alpha)$ comparison between BABAMC and BHLUMI we keep the same real emission distributions in both programs, i.e. that of BHLUMI (as in the first step), and we keep in each program the original distributions for the soft part. Here,

[6] Note that in the present exercise we use a variant of the SICAL trigger with an energy cut on Z_{min} and not on $U_{\min}$, where $U_{\min} < \min(E_1^{cl}, E_2^{cl})/E_{beam}$. The cut on Z_{min} is more appropriate for the present test on the hard bremsstrahlung matrix element.

the $\mathcal{O}(\alpha)$ Z boson contributions in BHLUMI are included according to eqs. (1) and this is exactly the actual implementation in the BHLUMI 4.02 version. The result of our comparison is shown in fig. 3, respectively for the real ALEPH SICAL detector[7] at energies below, at and above the Z peak, as a function of their respective energy cut variables. The difference between the two sets of results, those of BABAMC and those of BHLUMI, is below 0.005%. All the results for the Z boson contribution in the above pictures are shown as a fraction of the Born cross section for the pure t-channel γ exchange $\sigma_{\gamma_t \otimes \gamma_t}$.

We performed similar comparisons also for other values of CMS energy as given in tables 1–4 and for other cut parameters, similar to those described in ref. [19]. We have found that the difference between the BHLUMI and BABAMC results does not depend on the acceptance conditions but only slightly on the CMS energy value, but that in all cases it is below 0.005%.

We have also made the $\mathcal{O}(\alpha)$ comparisons of the kind presented in figs. 2 and 3 for the "academic" trigger of ref. [19]. We do not show explicitly results of this comparison due to the lack of space. More precisely we have compared the Z boson contribution including $\mathcal{O}(\alpha)$ QED corrections from BABAMC and BHLUMI for $|t_1| < |t| < |t_2|$, where $t_{1,2}$ at the Born level correspond to the SICAL angular acceptance of $1.6^\circ - 3.1^\circ$. We have obtained perfect agreement on $\sigma(V_{max})$ calculated from BABAMC and BHLUMI for all values of the energy cut parameter V_{max} in the energy cut $V_{max} > V$, where the variable V, defined in ref. [19], represents some kind of measure of the total energy carried away by all emitted real photons. The difference was always below 0.01%. This kind of trigger, in fact, cannot be realized in the real experiment, but it is useful for practical reasons[8]. We conclude that our new realization of the $\mathcal{O}(\alpha)$ Z_s contribution in BHLUMI 4.02 has been done with a *technical precision below* 0.005% practically for arbitrary experimental acceptance.

We now come to the most interesting part of our discussion, i.e. we are going to show our new results for δ_Z beyond $\mathcal{O}(\alpha)$ for two examples of the true (realistic) experimental acceptance. We shall compare BHLUMI 4.02 $\mathcal{O}(\alpha)$ results for δ_Z with and without YFS exponentiation for the ALEPH LCAL and SICAL detectors. Note that this is a definite improvement over refs. [8, 9] (see also table 4) where $\mathcal{O}(\alpha^2)$ ALIBABA and $\mathcal{O}(\alpha)$ BABAMC could be compared only for unrealistic acceptance. Our results will give us insight into size of corrections to δ_Z of higher order (mainly $\mathcal{O}(\alpha^2)$) in the real experiment.

In fig. 4 we show our predictions for δ_Z from BHLUMI 4.02 at the Born, $\mathcal{O}(\alpha)$ and YFS-exponentiated $\mathcal{O}(\alpha)$ levels, throughout the same energy regime as that considered in tables 1–4 for the ALEPH LCAL detector and in fig. 5 we show the analogous results for the ALEPH SICAL detector. (The other LEP collaborations have similar luminosity detectors in the similar angular ranges.) As we see, in the regime where the Born δ_Z is maximal in absolute value the size of the $\mathcal{O}(\alpha)$ correction is up to 50% of the Born value of δ_Z. The difference δ_Z'' corresponding to $\mathcal{O}(\alpha)^{YFS}_{exp} - \mathcal{O}(\alpha)$ is at most 0.15% for LCAL trigger and at most 0.03% for the SICAL trigger, in units of the Born cross section $\sigma_{\gamma_t \otimes \gamma_t}$. A

[7] We did the same comparison for the OPAL SiW acceptances getting the same good agreement.

[8] For this kind of acceptance important high-precision semi-analytical cross-checks of BHLUMI are feasible [19].

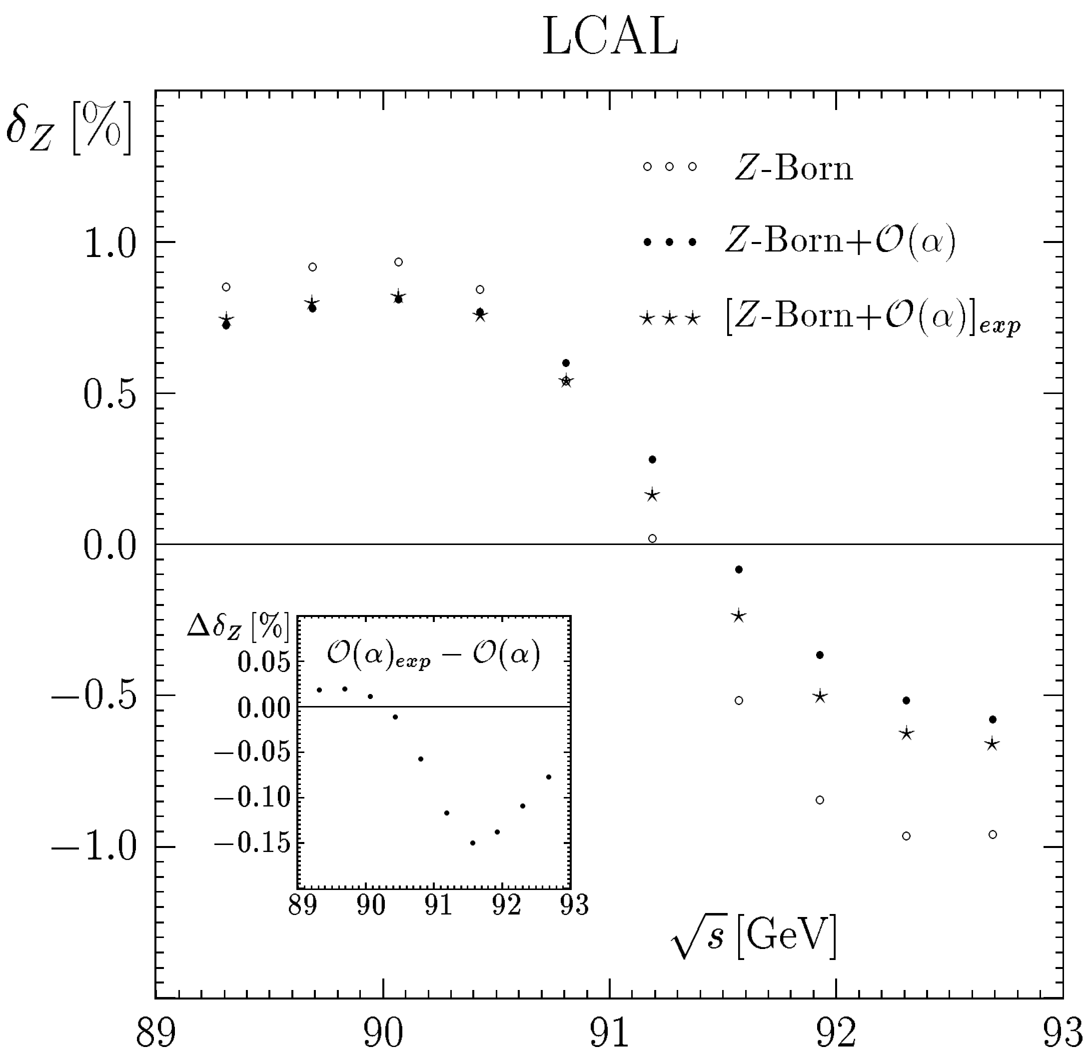

Figure 4: Z exchange contributions to low-angle Bhabha scattering for the LCAL acceptance with the energy cut $U_{min} = 0.5$ taken from BHLUMI, for $M_Z = 91.187\,\mathrm{GeV}$, $\Gamma_Z = 2.490\,\mathrm{GeV}$ and $\sin^2\theta_W = 0.2319$. The results are presented as a fraction (in %) of the Born cross section for the pure t-channel γ exchange. We used a simple MC model of the LCAL-type wide–narrow angular acceptance as described in detail in ref. [6] with the wide and narrow angular ranges $2.7^\circ < \theta < 7^\circ$ and $3.3^\circ < \theta < 6.3^\circ$, respectively. No acoplanarity cut was applied, i.e. the azimuthal size of each cluster was $0 \le \phi < 2\pi$.

conservative estimate of the uncertainty on δ_Z is to set it to largest difference δ_Z'' between $\mathcal{O}(\alpha)^{YFS}_{exp}$ and $\mathcal{O}(\alpha)$. For the LCAL and SICAL acceptances the conservative uncertainty would be 0.15% and 0.03% respectively, comparable to the uncertainties estimated in refs. [8, 9].

A more realistic estimate of the precision is the one which we now describe. It relies

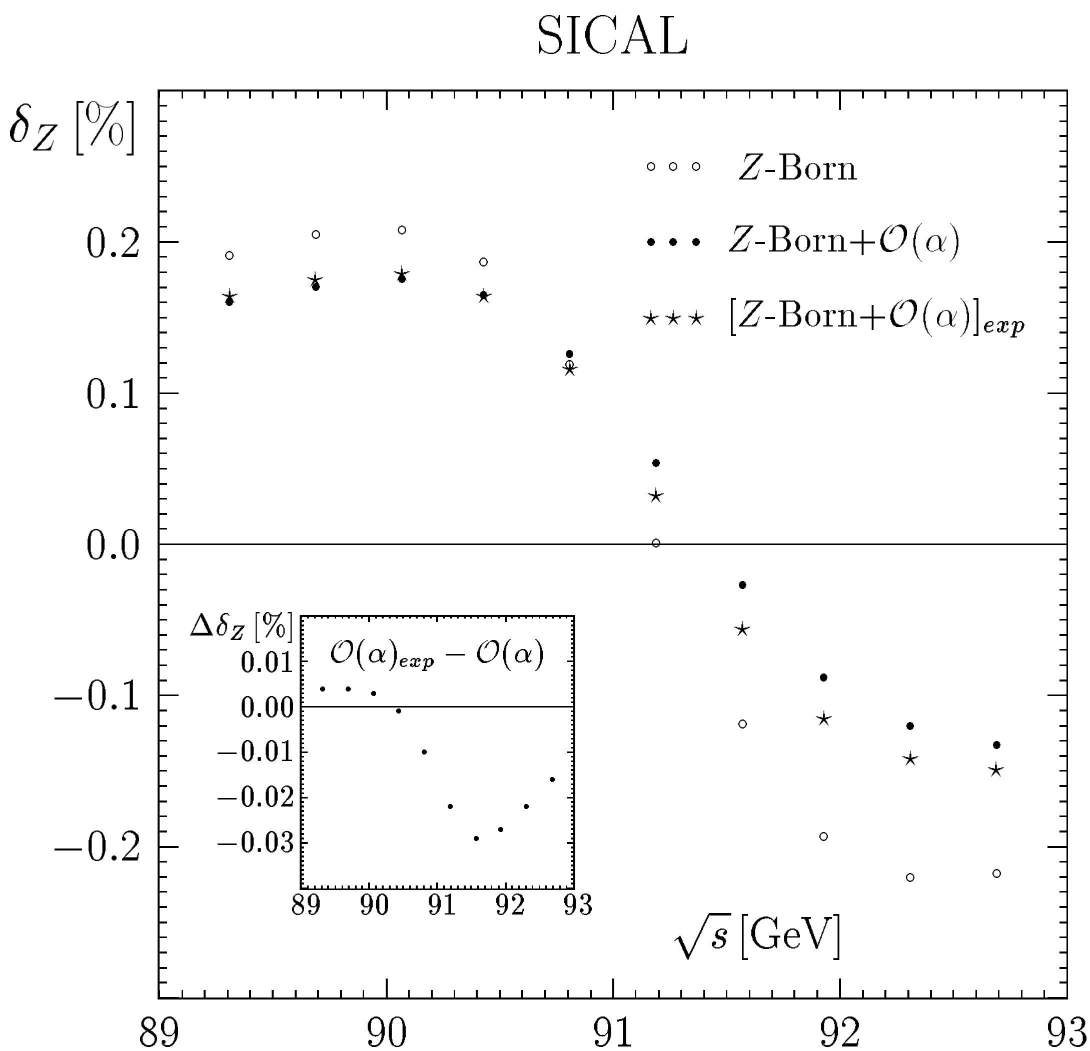

Figure 5: Z exchange contributions to low-angle Bhabha scattering for the SICAL acceptance with the energy cut $U_{min} = 0.43$ taken from BHLUMI, for $M_Z = 91.187\,\mathrm{GeV}$, $\Gamma_Z = 2.490\,\mathrm{GeV}$ and $\sin^2\theta_W = 0.2319$. The results are presented as a fraction (in %) of the Born cross section for the pure t-channel γ exchange.

on the observation (a) that we have quite good agreement of the $\mathcal{O}(\alpha)$ exponentiated BHLUMI with ALIBABA and on the observation (b) (already made in ref. [8]) that the difference between δ_Z calculated for *realistic* trigger such as the SICAL or LCAL and for the respective *simplified* non-calorimetric trigger without energy cut used in refs. [8, 9] is rather small. In the following we combine these two bits of information in a quantitative form. Here we understand that the $\mathcal{O}(\alpha)$ exponentiated δ_Z is subtracted from the luminosity measurement. In this case we have to estimate the total error of the cross section $\sigma(\mathcal{O})^{REAL}_{exp}$ for the *realistic* version of the trigger. We may express it as $\sigma\{\mathcal{O}(\alpha)_{exp}\}^{REAL} = \sigma\{\mathcal{O}(\alpha)\}^{REAL} + \Delta_1 + \Delta_2 - \Delta_3$, where Δ_i are defined uniquely in the following commutative graph

$$\begin{array}{ccc} \mathcal{O}(\alpha)^{SIMP} & \xrightarrow{\Delta_3} & \mathcal{O}(\alpha)^{REAL} \\ {\scriptstyle \Delta_1}\downarrow & & \downarrow \\ \mathcal{O}(\alpha)^{SIMP}_{exp} & \xrightarrow{\Delta_2} & \mathcal{O}(\alpha)^{REAL}_{exp} \end{array}$$

The observation (b) is quantified with $|\Delta_2 - \Delta_3| < 0.005\%$ for the LCAL trigger and $|\Delta_2 - \Delta_3| < 0.0007\%$ for the SICAL trigger, where the two numbers come from special high-statistics Monte Carlo runs. Now the total error we estimate as $\delta\sigma\{\mathcal{O}(\alpha)_{exp}\}^{REAL} = [(\delta\sigma\{\mathcal{O}(\alpha)\}^{REAL})^2 + (\delta\Delta_1)^2 + (\delta(\Delta_2 - \Delta_3))^2]^{1/2}$. For the error of $\delta(\Delta_2 - \Delta_3)$ we simply take the value of $\Delta_2 - \Delta_3$ itself. For $\delta\sigma\{\mathcal{O}(\alpha)\}^{REAL}$ we take its technical precision: 0.027% for LCAL (see table 2) and 0.006% for SICAL (see table 4). The physical and technical error $\delta\Delta_1$, the critical part in all this error accounting, we deduce from the comparison of the exponentiated BHLUMI versus $\mathcal{O}(\alpha^2)$ ALIBABA: 0.042% for LCAL (see table 2) and 0.007% for SICAL (see table 4). In this way we obtain the total error estimate (physical + technical) of 0.06% for the LCAL and 0.01% for the SICAL angular range. Since this error estimate depends heavily on the comparisons of our results with the ALIBABA code, we add one final consistency cross-check to the above procedure. In fig. 1 of ref. [20] we find the difference between the $\mathcal{O}(\alpha^2)$ ALIBABA and $\mathcal{O}(\alpha)$ BABAMC results. This difference should not be in contradiction, within the errors we have just obtained above, with the difference of the $\mathcal{O}(\alpha)_{exp}$ and $\mathcal{O}(\alpha)$ BHLUMI shown explicitly as insets in figs. 4 and 5. In the LCAL case the above two differences are however apart by more than 0.06%[9]. We could not find any immediate explanation for this discrepancy and in order to accommodate it in our error analysis we simply apply a safety factor 2 in $\delta\Delta_1$ for the LCAL case. This leads us to finally estimate the total error in the LCAL case to be 0.09%. In the case of the SICAL the situation is similar, and the same safety factor leads to a final error estimate of 0.015%. In future work, a more detailed analysis of the $\mathcal{O}(\alpha^2)$ corrections and of exponentiation is necessary to remove this safety factor. Finally let us note that the errors should scale smoothly for small differences in acceptance relative to the ones we have studied in detail. In particular the size of the errors of δ_Z should scale (as seen from comparing the SICAL and LCAL types of detectors) with the overall size of the δ_Z contribution. However, one should not extrapolate these results of the present paper to angular ranges above 6°. In particular it would be interesting to compare our results with the calculations and programs of the other groups presented in the recent LEP workshop on precision calculations for the Z resonance, see ref. [21].

We summarize our paper with the following conclusions:

- We have calculated the $\mathcal{O}(\alpha)$ YFS-exponentiated and pure $\mathcal{O}(\alpha)$ corrections to δ_Z, where the latter were based on ref. [14].

[9]Note that in view of the errors quoted in refs. [8, 9] such a discrepancy is acceptable.

- These results have been implemented in the Monte Carlo event generator BHLUMI and have been compared, in detail, with the work of refs. [8, 9]. More precisely, we have shown that we reproduce the $\mathcal{O}(\alpha)$ results of refs. [8, 9] for their non-calorimetric acceptance and we added more tests for examples of calorimetric realistic triggers. This is a very strong check that our $\mathcal{O}(\alpha)$ matrix element is correct.

- We also show that our $\mathcal{O}(\alpha)$ YFS-exponentiated results are within 0.007% of those predicted by ALIBABA, in the angular range 1.5°–3^0, for the non-calorimetric trigger amenable for ALIBABA.

- From the comparison of our $\mathcal{O}(\alpha)$ results with and without YFS exponentiation and also taking into account the successful comparison with the ALIBABA code, we deduce that in the angular range of about 1.5°–3^0, i.e. for the SICAL type of detector, the total precision of BHLUMI's new δ_Z correction is 0.015% (conservatively 0.03%). In the angular range of about 3°–6^0, i.e. for the LCAL type of detector, we estimate the total precision to be 0.09% (conservatively 0.15%).

- Users of BHLUMI are now armed with a new tool for calculating the δ_Z contribution and from now on they are not forced to use external codes to calculate it.

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