

# Exact $\mathcal{O}(\alpha)$ Gauge Invariant YFS Exponentiated Monte Carlo for (Un)Stable $W^+W^-$ Production At and Beyond LEP2 Energies<sup>†</sup>

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## Abstract

We realize, by Monte Carlo event generator methods, the exact  $\mathcal{O}(\alpha)$  YFS exponentiated calculation of  $e^+e^- \rightarrow W^+W^- (\rightarrow f_1 f'_1 + f_2 f'_2)$  at and beyond LEP2 energies, where the left-handed parts of  $f_i$  and  $f'_i$  are the respective upper and lower components of an  $SU_{2L}$  doublet,  $i = 1, 2$ . Our calculation is gauge invariant from the standpoint of its radiative effects and the respective YFS Monte Carlo event generator YFSWW3, wherein both Standard Model and anomalous triple gauge boson couplings are allowed, generates  $n(\gamma)$  radiation both from the initial state and from the final  $W^+W^-$ . Sample Monte Carlo data are illustrated.

*To be submitted to Physics Letters B*

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<sup>†</sup> Work partly supported by the Polish Government grants KBN 2P30225206 and 2P03B17210 and by the US Department of Energy Contracts DE-FG05-91ER40627 and DE-AC03-76ER00515.

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# 1 Introduction

The processes  $e^+e^- \rightarrow W^+W^- + n(\gamma) \rightarrow 4\text{fermions} + n(\gamma)$  at and beyond LEP2 energies are of considerable interest for the LEP2 and NLC physics programs. When calculated and measured to sufficient precision, they provide windows to several important avenues for verification and tests of the  $SU_{2L} \times U_1$  model (SM) of Glashow, Salam and Weinberg [1] of the electroweak interaction. In Ref. [2] for example, the goal for the theoretical precision of the understanding of these processes was set at  $\sim 0.5\%$  of the integrated cross section for the LEP2 physics program. In the following discussion, we will present calculations of these processes which feature the most precise electroweak radiative corrections which have been published to date, that of an exact  $\mathcal{O}(\alpha)$  YFS [3] exponentiated Monte Carlo event generator in which multiple photon radiative effects from both initial and  $W^+W^-$  states are realized on an event-by-event basis as infrared singularities are canceled to all orders in  $\alpha$ . We denote the respective event generator by YFSWW3. We will comment below on its physical and technical precision relative to the  $0.5\%$  integrated cross section precision goal of Ref. [2].

A few comments are in order regarding the relationship between the calculations presented below and those presented in Ref. [4]. In Ref. [4], we presented the second order leading-log YFS exponentiated calculation of the processes under discussion here and included for the first time the complete YFS formfactor effect for the radiation from the  $W^+W^-$  themselves, in a gauge invariant way [5, 6, 7]. One can recognize the procedure which we used in Ref. [4] to arrive at gauge invariant radiation from the  $W^+W^-$  themselves as an application of the fermion loop procedure expounded in some detail in Ref. [7]; indeed, the authors in this latter reference have shown that one arrives at amplitudes that are not only  $U_1$  invariant but also  $SU_{2L}$  invariant, guaranteeing the required gauge cancellations at high energies. With the advertised [2] precision goal of  $\sim 0.5\%$  for the integrated cross section for the final LEP2 theoretical precision tag on the signal prediction for the  $W^+W^-$  pair production, it is important to treat the complete  $\mathcal{O}(\alpha)$  exactly in the presence of our YFS exponentiation, including the exact pure weak corrections at  $\mathcal{O}(\alpha)$ , as these may enter at the level of this goal. It is for this reason that we are motivated to carry-out our work in this paper.

We indicate in our title that we consider in this paper the YFS exponentiation of the exact  $\mathcal{O}(\alpha)$  electroweak corrections to  $e^+e^- \rightarrow W^+W^-$  after which we allow the  $W^+W^-$  to decay according to their known branching fractions ( here, we use the SM Born level BR's). From the standpoint of the over-all process  $e^+e^- \rightarrow W^+W^- \rightarrow 4\text{fermions}$ , we compute the YFS exponentiated exact  $\mathcal{O}(\alpha)$  electroweak corrections for the initial state, initial-intermediate interference, and intermediate ( $W^+W^-$ ) state in the on-shell approximation as we describe further below. This means that we treat the so-called "CC3" class of doubly resonant signal graphs; for, we have in mind that, for example, for final states such as  $(q\bar{q}' + \mu\bar{\nu}_\mu)$ ,  $(q\bar{q}' + \tau\bar{\nu}_\tau)$ , a cut on the invariant masses  $M_{q\bar{q}'}$ ,  $M_{l\bar{\nu}_l}$  of an appropriate size (it should be  $\sim 60\text{GeV}$  at  $\sqrt{s} = 161\text{GeV}$ ) will be used to isolate the signal, CC3 graphs from the background graphs at the sub-per mille level (for the  $(q\bar{q}' + e\bar{\nu}_e)$

case, this cut needs to be  $\sim 78\text{GeV}$  at  $\sqrt{s} = 161\text{ GeV}$ <sup>1</sup>. In our over-all approach to precision LEP2  $WW$  physics, while we develop in this paper the Monte Carlo simulation of the YFS exponentiation of the exact  $\mathcal{O}(\alpha)$  electroweak corrections to the signal CC3 processes, we intend that the Monte Carlo KORALW [8] or a similar one as presented in Refs. [9, 10], with  $\mathcal{O}(\alpha^2)$  LL initial state radiation and the appropriate complement of background processes, is to be used in tandem with that developed herein for the desired level of simulation of the corresponding background processes. This will allow a complete picture of the  $WW$  pair production and decay with uncertainties at the level of  $\mathcal{O}(\frac{\alpha}{\pi} \frac{\Gamma_W}{M_W})$ . We only assert the physical precision (in the electroweak theory) up to uncertainties of  $\mathcal{O}(\frac{\alpha}{\pi} \frac{\Gamma_W}{M_W})$  because, as discussed in Refs. [11], there are initial-final, final-intermediate, and final-final state interference effects which are also relevant at this order for the inclusive distributions that we present in this paper. The extension of the results in Refs. [11] to exclusive distributions is under investigation and until this has been completed, we may only claim the electroweak physical precision estimate, for the combined predictions of our work in this paper and the results of KORALW for the background processes for example, of uncertainties at  $\mathcal{O}(\frac{\alpha}{\pi} \frac{\Gamma_W}{M_W})$  for inclusive distributions (the integration over *both* invariant masses of the  $W$ 's is done) with the understanding that the known  $W$  branching fractions are used to include the proper  $W$ -decay normalization. We also note that, recently, the authors in Refs. [12] have given a manifestly gauge invariant, independent and complete analysis of the non-factorizable effects analyzed in Refs. [11] and have arrived at entirely similar conclusions concerning when these corrections are significant. This provides an important cross-check on the results in Refs. [11], where we should note that the detailed formulas in Ref. [12] for the non-factorizable corrections themselves disagree with those in the second paper of Ref. [11].

Specifically, what we will do is the following. In the next section, we shall describe the extension the analysis in Ref. [4] to include final state multiple photon radiation at the leading-log (LL) level. This will lead to the development of the new YFS exponentiated Monte Carlo event generator YFSWW3, involving as it will the extension of the YFS3 event generator in Ref. [13] for the process  $e^+e^- \rightarrow f\bar{f} + n(\gamma)$  to the process  $e^+e^- \rightarrow W^+W^- + n(\gamma)$ , where the  $W$ -pair is then allowed to decay. This will set the stage for our work in section 3, wherein we will include in the YFSWW3 event generator the exact  $\mathcal{O}(\alpha)$  results for the hard photon residuals  $\bar{\beta}_i, i = 0, 1$  in the language of Ref. [3] (this language will be reviewed briefly below) basing ourselves on the exact virtual electroweak  $\mathcal{O}(\alpha)$  results and the exact  $\mathcal{O}(\alpha)$  bremsstrahlung results of Refs. [14]. Entirely equivalent results on the exact electroweak  $\mathcal{O}(\alpha)$  corrections to  $W$ -pair production have also been given in Refs. [15, 16]. We note that the exact  $\mathcal{O}(\alpha)$  electroweak correction to the  $W$  decay width have also been given in the literature in Refs. [17] but that, as we only treat the exact electroweak  $\mathcal{O}(\alpha)$  corrections to the  $W$ -pair production in this paper, we will not need these latter results here – they will be taken-up elsewhere [18] when we consider the exact YFS exponentiated electroweak  $\mathcal{O}(\alpha)$   $W$ -decay distributions. In section 4, we will then present some sample Monte Carlo data to illustrate the size of the various levels of

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<sup>1</sup>Our background reduction cuts were determined using the Monte Carlo KORALW [8].

approximation that we access in our work, with some focus on what is currently available in the literature and how our results relate to it. Section 5 contains some concluding remarks.

## 2 Multiple Initial + Final State Photon Radiative Effects

In this section we discuss the relevant aspects of our YFS Monte Carlo methods as they pertain to the problem of extending our YFS3 MC in Ref. [13] for the processes  $e^+e^- \rightarrow f\bar{f} + n(\gamma)$  in which both initial and final state YFS exponentiated multiple photon radiation is simulated to the  $W^+W^-$  pair production processes of interest to us here. We carry-out the respective extension as well to arrive at a new MC event generator, YFSWW3, in which not only is the full YFS formfactor effect with the  $W$ -pair contribution included as in YFSWW2 [4] but also real soft final state  $n(\gamma)$  radiation from the  $W$ -pair itself is calculated. Such results have not appeared elsewhere. On the way we also set our notation and define our kinematics for the work in the remaining sections.

Specifically, the processes of interest to us here are illustrated in Fig. 1, where we have given our kinematics.

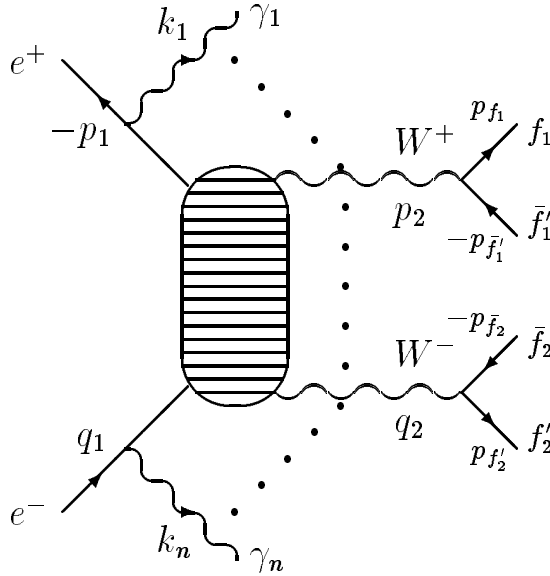


Figure 1: The process  $e^+e^- \rightarrow W^+W^- \rightarrow 4fermions = f_1 + \bar{f}'_1 + \bar{f}_2 + f'_2$ , where  $\begin{pmatrix} f_i \\ f'_i \end{pmatrix}_L \equiv \frac{1}{2}(1 - \gamma_5) \begin{pmatrix} f_i \\ f'_i \end{pmatrix}$ ,  $i = 1, 2$ , are  $SU_{2L}$  doublets. Here,  $p_A$  is the 4-momentum of  $A$ ,  $A = f_i, f'_i, p_1(q_1)$  and  $p_2(q_2)$  are the 4-momenta of  $e^+(e^-)$  and  $W^+(W^-)$  respectively.

We consider  $e^+ + e^- \rightarrow W^+ + W^- + n(\gamma) \rightarrow 4fermions + n(\gamma)$  at CMS energies  $\sqrt{s} \geq 2M_W$ . In Ref. [4], we have shown that our YFS exponentiated Monte Carlo algorithms in Refs. [19, 13] for the processes  $e^+e^- \rightarrow f + \bar{f} + n(\gamma)$ , where  $f$  is a fundamental fermion in the  $SU_{2L} \times U_1$  theory [1], has a gauge invariant extension to  $e^+ + e^- \rightarrow W^+ + W^- + n(\gamma) \rightarrow 4fermions + n(\gamma)$ . In Ref. [4] we presented the extension for our YFS2 initial state multiple photon Monte Carlo as the YFSWW2 multiple photon Monte Carlo in which the full YFS form factor was taken into account. Here, we present the corresponding extension of our YFS3 initial + final state multiple photon Monte Carlo in which multiple photon radiation from the  $W$ 's themselves is included in the lowest level of the Monte Carlo algorithm. We refer to this extension of YFS3 to the  $W$ -pair production process as YFSWW3 and it is available from the authors [20].

Specifically, on applying the results in Ref. [4] for extending, to  $W$ -pair production in a gauge invariant way, our YFS formula for the process for  $e^+e^- \rightarrow f + \bar{f} + n(\gamma)$  to our formula for the cross section in Ref. [13], we arrive at the gauge invariant YFS formula for the respective  $W$ -pair production cross section in which multiple initial state and multiple final state radiation is calculated, where we use the ratio of  $\Gamma_W/M_W$  to justify treating the  $W$ -pair state as a “final” state on their mass shell insofar as these radiative effects are concerned. We then realize this formula by Monte Carlo methods in complete analogy with what is done in Refs. [4, 13], using as our Born cross section that given by Ref. [21]. The resulting Monte Carlo, YFSWW3, at high energies compared to  $M_W$  features final state leading-log  $\mathcal{O}(\alpha^2)$  radiative effects as well as the initial state leading-log  $\mathcal{O}(\alpha^2)$  radiative effects in YFSWW2. Thus, at NLC energies, these final state radiative effects may be important as we shall illustrate presently. In addition, the YFSWW3 final state  $n(\gamma)$  radiation allows us to check the technical precision of the  $\bar{\beta}_0$ -level  $n(\gamma)$  soft radiation in YFSWW2 since the final state  $n(\gamma)$  radiation is already present in the low level background Monte Carlo in YFSWW3 whereas in YFSWW2 it is not. We will also illustrate this presently.

$E_{CM} [GeV]$	ISR	+ Coul. corr.	+ $Y'$ -corr.	+ $WW$ -rad.
175	$0.4908 \pm 0.0002$	$0.5047 \pm 0.0002$	$0.5056 \pm 0.0002$	$0.5056 \pm 0.0002$
	$0.4900 \pm 0.0002$	$0.5039 \pm 0.0002$	$0.5052 \pm 0.0002$	$0.5051 \pm 0.0002$
190	$0.6067 \pm 0.0008$	$0.6198 \pm 0.0009$	$0.6224 \pm 0.0009$	$0.6224 \pm 0.0009$
	$0.6039 \pm 0.0008$	$0.6169 \pm 0.0008$	$0.6200 \pm 0.0009$	$0.6202 \pm 0.0009$
205	$0.6362 \pm 0.0009$	$0.6475 \pm 0.0009$	$0.6513 \pm 0.0010$	$0.6525 \pm 0.0010$
	$0.6316 \pm 0.0009$	$0.6429 \pm 0.0009$	$0.6472 \pm 0.0010$	$0.6484 \pm 0.0010$
500	$0.2912 \pm 0.0003$	$0.2943 \pm 0.0003$	$0.2973 \pm 0.0004$	$0.2975 \pm 0.0004$
	$0.3535 \pm 0.0004$	$0.3574 \pm 0.0004$	$0.3587 \pm 0.0005$	$0.3597 \pm 0.0005$

Table 1: The results of the  $10^5$  (except for  $E_{CM} = 175 GeV$ , where it is  $10^6$ ) statistics sample (unweighted events) from YFSWW3 for the total cross section  $\sigma [pb]$ . The upper results at each value of energy are for the Standard Model couplings constants, while the lower ones are for anomalous couplings constants ( $\delta\kappa = \delta\lambda = 0.1$ ). See the text for more details.

More precisely, in Table 1, we show the result of our YFSWW3.0 simulation of the process in Fig. 1 with the  $c\bar{s} + e\bar{\nu}_e$  final state, for CMS energies 175 GeV, 190 GeV, 205 GeV and 500 GeV which have LEP2 and NLC in mind. We present results according to the notation in Ref. [4] so that ISR denotes initial state  $n(\gamma)$  radiation, “Coul. corr.” denotes that the Coulomb correction after the fashion of the first paper in Ref. [22] is included, “Y'-corr.” denotes that the full YFS form-factor effect already featured in Ref. [4] is turned on, and finally “WW-rad.” denotes the new feature of YFSWW3.0 in which the real soft  $n(\gamma)$  bremsstrahlung from the  $W$ -pair is included in the simulation (already at the level of the low level Monte Carlo algorithm in the sense of Ref. [19] for example). In addition, the upper and lower results in each entry in the table correspond to the case of Standard Model and anomalous ( $\delta\kappa = \delta\lambda = 0.1$ )  $WWV$  couplings in the notation of Ref. [21]. Thus, the comparison of the last two columns shows that the effects of the final state  $n(\gamma)$ , which are expected to be negligible at LEP2 energies, are at the  $\sim 2$  per mille level ( $\sim$  twice our statistical error) at NLC energies. Comparison of the first three columns in the table with the analogous results in the respective Table 1 of Ref. [4] shows that indeed we have a very good agreement between YFSWW2 and YFSWW3.0 in the calculation of effects in which the real final state radiation should not be important (the agreement is below the  $\sim$  per mille level statistical error on the respective results with the exception of the last entry in column 2, where the agreement is at the 2 per mille level). These comparisons represent a very good technical precision check on the two calculations. Moreover, they show that YFSWW3.0 is an excellent starting point for developing the exact  $\mathcal{O}(\alpha)$  YFS exponentiated  $\bar{\beta}_1$ -level Monte Carlo event generator calculation in which one has an exact  $\mathcal{O}(\alpha)$  calculation in the presence of initial + final state ( $W^+W^-$ )  $\mathcal{O}(\alpha^2)$  leading-log radiation with YFS exponentiation – such a calculation has not appeared elsewhere. To this we now turn in the next section.

### 3 Exact $\mathcal{O}(\alpha)$ YFS Exponentiated WW-Pair Production: YFSWW3.1

In this section, we develop the exact  $\mathcal{O}(\alpha)$  realization of the hard photon residuals  $\bar{\beta}_n, n = 0, 1$  in our YFSWW3 Monte Carlo event generator, where we refer to Refs. [3, 19, 4] for a precise definition of these residuals. We start with  $\bar{\beta}_0$ .

For the construction of the exact  $\mathcal{O}(\alpha)$  electroweak result for  $\bar{\beta}_0$ , which we denote by  $\bar{\beta}_0^{(1)}$ , we first note that

$$\frac{1}{2}\bar{\beta}_0^{(1)} = d\sigma^{one-loop}/d\Omega - 2\alpha\Re B d\sigma_B/d\Omega \quad (1)$$

where the YFS virtual infrared function  $B$  for the process  $e^+ + e^- \rightarrow W^+ + W^-$  is defined in Ref. [4], where the cross section  $d\sigma_B/d\Omega$  is the respective Born cross section, and where  $d\sigma^{one-loop}/d\Omega$  is the respective exact one-loop correction to the cross section given by the results of Ref. [14]. We have implemented the formula in (1) in our YFSWW3 Monte

Carlo event generator to arrive at a realization of the exact  $\mathcal{O}(\alpha)$  electroweak result for the hard photon residual  $\bar{\beta}_0$ .

Turning now to the exact  $\mathcal{O}(\alpha)$  result for the hard photon residual  $\bar{\beta}_1$ , which we denote by  $\bar{\beta}_1^{(1)}$ , we first note that

$$\frac{1}{2}\bar{\beta}_1^{(1)} = d\sigma^{B1}/kdkd\Omega_\gamma d\Omega - \tilde{S}(k)d\sigma_B/d\Omega \quad (2)$$

where  $\tilde{S}(k)$  is the respective YFS real infrared function defined in Ref. [4] and  $d\sigma^{B1}/kdkd\Omega_\gamma d\Omega$  is the respective exact  $\mathcal{O}(\alpha)$  bremsstrahlung cross section<sup>2</sup> from Ref. [14] for the photon into the phase space element  $kdkd\Omega_\gamma$  when the  $W^-$  is produced into the solid angle  $d\Omega$ , for example. We have implemented the result (2) into our YFSWW3 Monte Carlo as well and the resulting version of it, including both the results in (1) and in (2) is version 3.1: YFSWW3.1. In the next section, we illustrate some of its applications.

Before turning to these applications, let us comment more explicitly on the actual implementation of (1) and (2) in our Monte Carlo YFSWW3.1. As we have noted above, we are using the fermion loop scheme methodology [7] to maintain gauge invariance of the  $W$ -pair radiation. Thus, we are also using this scheme to organize our exact  $\mathcal{O}(\alpha)$  corrections in  $\bar{\beta}_n$ ,  $n = 0, 1$  as well. To make contact from the off-shell  $e^+e^- \rightarrow W^+W^- + n(\gamma) \rightarrow 4fermions + n(\gamma)$  point to the on-shell  $e^+e^- \rightarrow W^+W^- \rightarrow 4fermions$  point needed for  $\bar{\beta}_0$  and to the on-shell  $e^+e^- \rightarrow W^+W^- + \gamma \rightarrow 4fermions + \gamma$  point needed for  $\bar{\beta}_1$ , we first use our usual YFS reduction procedure (see Ref. [4] and references therein for an explicit description of this procedure) to reach the respective off-shell points for the  $\bar{\beta}_n$ ,  $n=0,1$ ; then, to use the respective on-shell exact  $\mathcal{O}(\alpha)$  results in Ref. [14], we identify the respective off-shell phase space point with the corresponding one for the on-shell point with the same CMS relative velocity and CMS scattering angle – this latter on-shell point is used to evaluate the respective exact  $\mathcal{O}(\alpha)$  corrections and, very generally, we expect our procedure for using these corrections to be accurate up to uncertainties of  $\mathcal{O}(\frac{\Gamma_W}{M_W} \frac{\alpha}{\pi})$ .

We turn now to applications of YFSWW3.1.

## 4 Illustrative Results from YFSWW3.1

In this section we illustrate the application of the exact  $\mathcal{O}(\alpha)$  YFS exponentiated Monte Carlo event generator YFSWW3.1 to the  $W$ -pair production and decay at LEP2 and NLC energies, where as we have explained above by “ $\mathcal{O}(\alpha)$ ” we refer to the  $\mathcal{O}(\alpha)$  electroweak corrections to the  $W$ -pair production (initial, initial-intermediate, and intermediate state corrections); for definiteness, henceforth, we denote this set of corrections with the notation “ $\mathcal{O}(\alpha)_{prod}$ ”. We continue to use the  $e\bar{s} + e\bar{\nu}_e$  4-fermion final state for definiteness.

Specifically, we have recorded in Table 2 a summary of the sizes of the various approximations which we have realized in YFSWW3.1 including the exact  $\mathcal{O}(\alpha)_{prod}$  YFS exponentiated result. We show results for CMS energies  $E_{CM}$  of 161, 175, 190, 205 and

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<sup>2</sup>We would like to thank K. Kołodziej for providing us with routines for the  $\mathcal{O}(\alpha)$  hard bremsstrahlung matrix element.

$E_{CM} [GeV]$	$\sigma_0 [pb]$	$(\sigma_2^{prag} - \sigma_0)/\sigma_0$	$(\sigma_2^{LL} - \sigma_1^{LL})/\sigma_0$	$(\sigma_1^{ex} - \sigma_1^{LL})/\sigma_0$	$(\sigma_1^{ex} - \sigma_1^{ap})/\sigma_0$
161	0.1768	-24.6%	+0.11%	-0.83%	+0.22%
175	0.5891	-15.4%	+0.12%	-1.32%	-0.006%
190	0.6792	-9.9%	+0.11%	-1.71%	-0.22%
205	0.6850	-6.8%	+0.10%	-2.22%	-0.61%
500	0.2710	+5.1%	+0.19%	-4.72%	-3.09%
		+5.1%	+0.19%	-5.16%	-3.09%

Table 2: Various contributions (approximations) to the YFS exponentiated  $WW$  cross section as a fraction [in %] of the Born cross section  $\sigma_0$ :  $\sigma_2^{prag}$  denotes the so-called pragmatic  $\mathcal{O}(\alpha^2)$  cross section (see the text for more details),  $\sigma_1^{LL}$  and  $\sigma_2^{LL}$  are the LL approximations of  $\mathcal{O}(\alpha^1)$  and  $\mathcal{O}(\alpha^2)$ , respectively,  $\sigma_1^{ex}$  is the exact  $\mathcal{O}(\alpha^1)$  result, while  $\sigma_1^{ap}$  is the so-called Improved Born Approximation (see the text for more details). The results in the lower line for  $E_{CM} = 500 GeV$  correspond to the situation when LL QED corrections for the  $W^+W^-$  state are also included in the corresponding LL contributions.

500 GeV. The definitions of the YFS exponentiated cross sections in the table are as follows:  $\sigma_2^{prag}$  denotes that the cross section contains the exact  $\mathcal{O}(\alpha)$  correction and the  $\mathcal{O}(\alpha^2)$  initial state LL correction at the level of the  $W$ -pair production process;  $\sigma_n^{LL}$  denotes that the cross section contains initial state LL QED radiative corrections through  $\mathcal{O}(\alpha^n)$ ;  $\sigma_n^{ex}$  denotes that the cross section contains the exact radiative correction through  $\mathcal{O}(\alpha^n)_{prod}$ ;  $\sigma_n^{ap}$  indicates that the cross section contains an approximate treatment of the radiative corrections through  $\mathcal{O}(\alpha^n)_{prod}$  (so-called Improved Born Approximation) and we only discuss the case  $n = 1$  where we have implemented the approximate cross section of Ref. [23]. The second entry for  $E_{CM} = 500 GeV$  corresponds in each column to a different treatment of the LL QED radiation in which both initial and final state LL QED effects are included – it gives us an estimate of the size of the sub-leading QED radiative effects at the higher orders when it is compared with the first entry for example. The cross sections shown in the table correspond to simulations of  $10^6$  (weighted) events at all energies except for the  $E_{CM} = 500 GeV$  results, where  $10^7$  events were simulated. What we see in the table is that the  $\mathcal{O}(\alpha^2)$  LL effects are well below the targeted 0.5% (of the integrated cross section) precision tag of the Physics at LEP2 “WW Cross-Sections and Distributions” Physics Working Group [2] for all LEP2 energies. For the 500 GeV case, the high precision objectives of certain NLC physics issues would clearly necessitate inclusion of these second order LL effects. The difference between the exact  $\mathcal{O}(\alpha)_{prod}$  result  $\sigma_1^{ex}$  and the first order LL result  $\sigma_1^{LL}$  shows that even at LEP2 energies, the non-leading QED and pure EW corrections are important [23]; and, as advertised in Ref. [23], the use of the approximation  $\sigma_1^{ap}$  at LEP2 energies would give results within the desired 0.5% integrated cross section precision tag except perhaps at the highest LEP2 CMS energy of 205 GeV, where the difference between the exact result and the  $\sigma_1^{ap}$  is 0.61%, just beyond the desired tag. Thus, for all but the highest LEP2 energies, the  $\sigma_1^{ap}$  could be used to realize a gain in CPU time for event generation of a factor of  $\sim 30$  without an unacceptable loss in precision relative a 0.5% integrated cross section precision tag. At the NLC type

energy  $E_{CM} = 500$  GeV the approximation  $\sigma_1^{ap}$  is not supposed to be accurate and our last column bears this out, although we see that it is better than the  $\sigma_1^{LL}$  at such high energies. We are encouraged that our best result,  $\sigma_2^{prag}$ , is not very sensitive to the inclusion of the final state LL QED ( $\alpha^2$ ) effects at 500 GeV; this is not true of  $\sigma_1^{LL}$ , as expected. We stress that, in the final state,  $L = \ln(s/M_W^2) - 1$  is only 2.67 for  $E_{CM} = 500$  GeV so that the LL series in the final state is one of several effects of comparable significance in comparison to the LL initial state series at ( $\alpha^2$ ) and we show the last row in the table to illustrate the size of such effects.

$E_{CM} [GeV]$	$(\bar{\beta}_0^{(1)} - \bar{\beta}_{0,LL}^{(1)})/\sigma_0$	$(\bar{\beta}_1^{(1)} - \bar{\beta}_{1,LL}^{(1)})/\sigma_0$
161	-0.84%	+0.02%
175	-1.24%	-0.08%
190	-1.58%	-0.14%
205	-2.06%	-0.16%
500	-5.41%	+0.69%
	-6.09%	+0.97%

Table 3: Differences between exact and LL approximated non-infrared contributions to the YFS exponentiated  $WW$  cross section as a fraction [in %] of the Born cross section  $\sigma_0$ . The results in the lower line for  $E_{CM} = 500$  GeV correspond to the situation when LL QED corrections for the  $W^+W^-$  state are also included in the corresponding LL contributions.

Since we have incorporated the exact  $\mathcal{O}(\alpha)_{prod}$  correction in our calculation, it is instructive to look at the size of its effect in comparison to the LL QED radiative correction insofar as our YFS hard photon residuals are concerned. We show this in Table 3. In the table, the notation is such that  $\bar{\beta}_n^{(m)}$  is the  $\mathcal{O}(\alpha^m)_{prod}$  result for the hard YFS photon residual  $\bar{\beta}_n$  and the subscript  $LL$  denotes that it is computed in the LL approximation (in the initial state LL approximation to be precise, except in the 500 GeV results where in the last row  $WW$  LL radiation is also included). What we see in the table is that the main effect of the exact result in comparison to the LL one is realized in the hard photon residual  $\bar{\beta}_0$ ; the effect on  $\bar{\beta}_1$  in comparison to the LL approximation is well below the 0.5% integrated cross section precision tag of Ref. [2] for LEP2 energies. For the NLC energy 500 GeV, we see that, while the main difference between the exact result and the LL approximation is still in  $\bar{\beta}_0$ , that in  $\bar{\beta}_1$  is also significant for high precision NLC physics objectives.

These results on the total cross sections are then analyzed in more detail in Figs. 2, 3, 4, wherein we show the  $W^-$  angular distributions corresponding to the respective  $E_{CM}$  values in Tables 1-3. For each value of  $E_{CM}$  we show the  $W^-$  angular distribution at the Born level and at the  $\mathcal{O}(\alpha^2)_{exp}^{prag}$  level ( $\sigma_2^{prag}$  level) as well as the differences  $\Delta d\sigma(\theta)_{2-1} = d\sigma_2^{LL}(\theta) - d\sigma_1^{LL}(\theta)$ ,  $\Delta d\sigma(\theta)_{ex-1} = d\sigma_1^{ex}(\theta) - d\sigma_1^{LL}(\theta)$ , and  $\Delta d\sigma(\theta)_{ex-ap} = d\sigma_1^{ex}(\theta) - d\sigma_1^{ap}(\theta)$  in ratio to the respective Born level differential cross section  $d\sigma_0(\theta)$ , where Fig. 1 contains this information for  $E_{CM} = 161, 175$  GeV, Fig. 2 contains it for  $E_{CM} = 190, 205$  GeV, and Fig. 3 contains it for  $E_{CM} = 500$  GeV. In the figures, we denote by  $\mathcal{O}(\alpha^n)_{exp}^{LL}$  the YFS

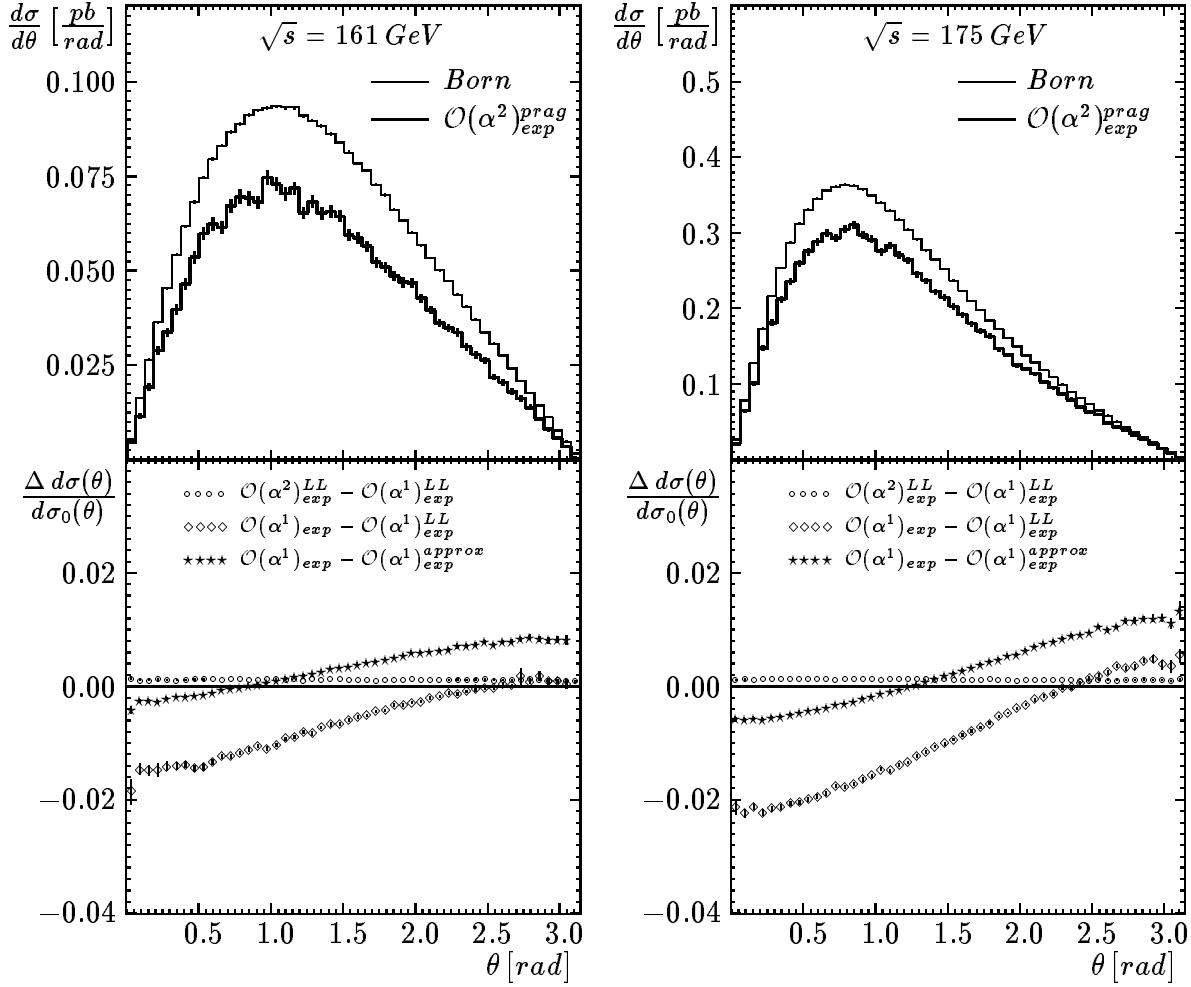


Figure 2: The  $W^-$  angular distributions for two LEP2 energies:  $\sqrt{s} = 161$  GeV (left picture) and  $\sqrt{s} = 175$  GeV (right picture). The upper parts of the pictures show the differential cross sections in Born (thin-lines) and in  $\mathcal{O}(\alpha^2)_{exp}^{prag}$  (thick-lines) approximations, where the latter one denotes the YFS exponentiated cross section including the exact  $\mathcal{O}(\alpha^1)$  matrix element together with the 2-nd order LL ISR terms (see the text for more details). In the lower part of each picture the following contributions to the YFS exponentiated cross section (divided by the corresponding Born differential cross section  $d\sigma_0$ ) are presented: the 2-nd order LL ISR contribution ( $\circ\circ$ ), the 1-st order non-LL contribution including electroweak radiative corrections ( $\diamond\diamond$ ) and the difference between the exact  $\mathcal{O}(\alpha^1)$  matrix elements and the so-called Improved Born Approximation ( $\star\star$ ). The last one shows the quality of IBA for the differential cross section as a function of the  $W$ -boson polar angle.

exponentiated cross section  $\sigma_n^{LL}$  level, by  $\mathcal{O}(\alpha^n)_{exp}$  the YFS exponentiated cross section  $\sigma_n^{ex}$  level, and by  $\mathcal{O}(\alpha^n)_{exp}^{approx}$  the YFS exponentiated cross section  $\sigma_n^{ap}$  level. The differential cross section results are fully consistent with the total cross section results shown above. Moreover, they show us that at all energies, the radiative corrections are significant

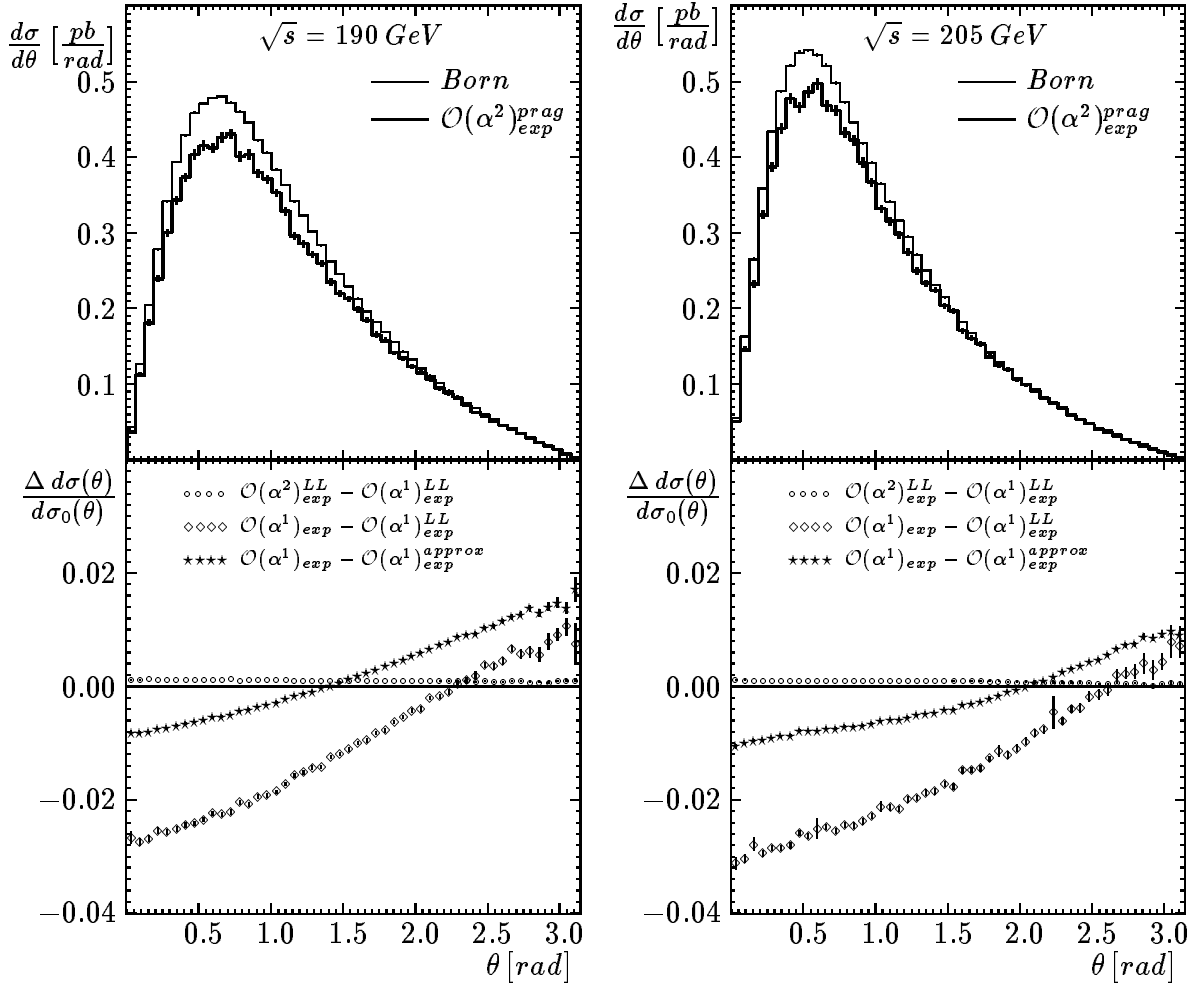


Figure 3: The same as in Fig. 2 but for other LEP2 energies:  $\sqrt{s} = 190$  GeV (left picture) and  $\sqrt{s} = 205$  GeV (right picture).

throughout the entire angular distribution. At LEP2 energies, the angular dependence of the ratio  $\Delta d\sigma(\theta)_{2-1}/d\sigma_0(\theta)$  is essentially flat; at the NLC energy 500 GeV, it is non-trivial and strongly varying in the backward direction. The ratios  $\Delta d\sigma(\theta)_{ex-1}/d\sigma_0(\theta)$  and  $\Delta d\sigma(\theta)_{ex-ap}/d\sigma_0(\theta)$  are smoothly varying in  $\theta$  at LEP2 and at NLC energies; at all energies, in the region where the cross sections are largest, the latter is smaller in magnitude than the former, as expected; away from this region, especially near the backward direction, the opposite is generally true. Of course, at the NLC energy 500 GeV, one does not expect the cross section  $\sigma_1^{ap}$  to be as accurate as it is at the LEP2 energies for which it was optimized and this is borne-out both in the  $\theta$  dependence of the ratio  $\Delta d\sigma(\theta)_{ex-ap}/d\sigma_0(\theta)$  as well as in the total cross section results in Table 2. As discussed in Refs. [24], this deviation of  $\sigma_1^{ap}$  from the exact  $\mathcal{O}(\alpha)_{prod}$  results at NLC energies is due mainly to corrections

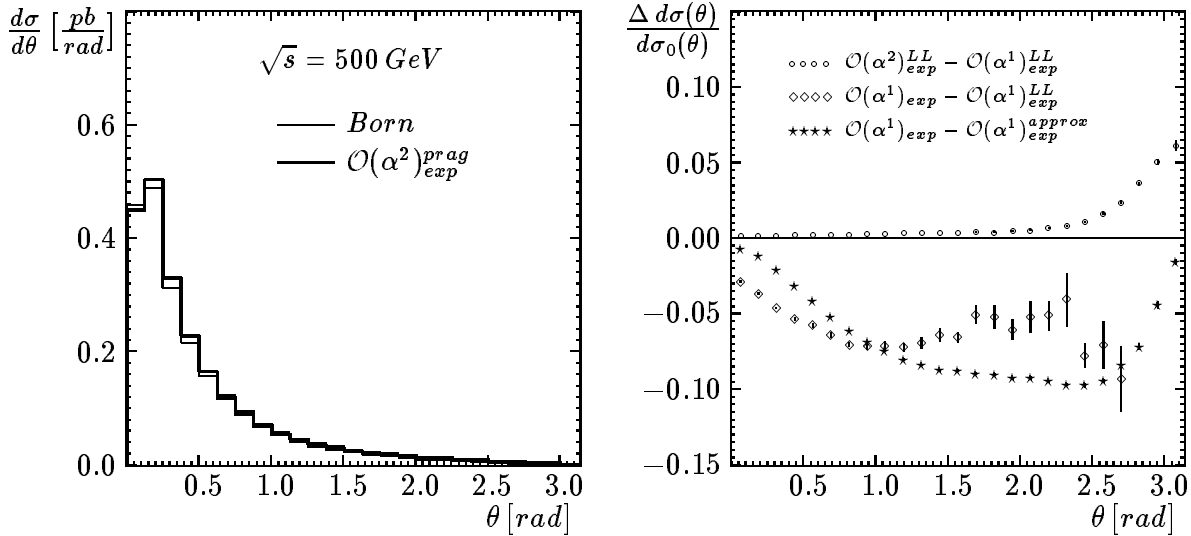


Figure 4: The same as in Fig. 2 but for the NLC energy:  $\sqrt{s} = 500$  GeV.

of order  $\frac{\alpha}{\pi} \ln^2(s/M_W^2)$  which are not part of the usual improved Born approximation – for further discussion of this point see Refs. [25] wherein the effects are explicitly calculated. Not shown in the Figs. 2-4 are the corresponding results for the initial + final state LL YFS exponentiated approximation at 500 GeV; its curves are entirely similar to those shown in Fig. 4 so that we do not present them separately here.

Our conclusion is that the exact results show that for the targeted precision of the Physics at LEP2 Workshop the LL approximation is inadequate, both for the total cross section and for the differential cross section; the non-leading corrections must be taken into account. In YFSWW3.1, we have the only multiple photon YFS exponentiated amplitude based calculation of these effects in which the respective infrared singularities are canceled to all orders in  $\alpha$  and in which the corrections themselves are realized on an event-by-event basis.

## 5 Conclusions

In this paper we have presented and illustrated the first ever 4-fermion Monte Carlo event generator for  $W^+W^-$  pair production and decay in which the full EW  $\mathcal{O}(\alpha)_{prod}$  correction is calculated in the presence of amplitude based YFS exponentiation in which infrared singularities are canceled to all orders in  $\alpha$ . The resulting Monte Carlo event generator, YFSWW3.1, is available from the authors [20].

As we have illustrated by showing the results of simulations at several LEP2 energies and at the NLC energy 500 GeV, the nonleading  $\mathcal{O}(\alpha)_{prod}$  EW corrections are indeed important and must be taken into account, both for the overall normalization and for the detailed angular distributions. At LEP2 energies, we showed that the approximate

representation in Ref. [23] of these non-leading corrections does indeed stay within 0.61% of the exact  $\mathcal{O}(\alpha)_{prod}$  EW integrated cross section result in the presence of our YFS exponentiation as we realize it by Monte Carlo methods; for the NLC energy, this is no longer the case, as expected. We stress that these last remarks apply both to the total cross sections and to the differential cross sections in the regions where they have their largest values. Our  $\bar{\beta}_1$  level second order LL YFS exponentiated results are thus a significant improvement over the  $\bar{\beta}_0$  level results in Ref. [4]; it is also a significant improvement for the signal CC3 processes over the other available Monte Carlo event generators [9, 10] in the literature; for, none of them feature a 4-fermion, amplitude based, exact EW  $\mathcal{O}(\alpha)_{prod}$  YFS exponentiated calculation in which infrared singularities are canceled to all orders in  $\alpha$ , on an event-by-event basis as we do in YFSWW3.1. We look forward with excitement to the many applications of our work.

## Acknowledgements

Two of us (S. J. and B.F.L. W.) acknowledge the kind hospitality of Prof. G. Veneziano and the CERN Theory Division while this work was completed. One of us (B.F.L.W.) acknowledges the support of Prof. D. Schlatter and the ALEPH Collaboration in the final stages of this work.

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